Wave Propagation Solution for Transverse Electric Mode in a Graded Interface between Left-Handed and Right-Handed Material

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Abstract

Wave propagation for transverse electric (TE) mode in a graded interface between left-handed and right-handed material has been investigated by using asymptotic iteration method. By using hyperbolic functions for negative permittivity and negative permeability, we obtained the graded graphs of permittivity and permeability as a function of material thickness. Maxwell equation for the dielectric with the hyperbolic function in permittivity and permeability has been reduced to second orde differential equation. The second orde differential equation has been solved by using asymptotic iteration method with the eigen functions for the fields equation in polynomial Hermite. The eigen functions explained about the wave propagation in a graded interface of material. The distribution of the electric field and the wave vector were given in approximate solution.

1. Introduction

The development of physics related to artificial composite material has been brought into a new finding, namely a negative refractive index material (NIM) or so-called lefthanded materials (LHM). NIM or LHM is better known as a metamaterial. For more than ten years, the study of the metamaterial has attracted the interest of many scientists.[1-2] Various studies, either theoretically or experimentally has been done by many scientists. The first time, in 1968, Vaselago reviewing theoretically what if a material is based on the negative electric permittivity and negative magnetic permeability.[3] Meanwhile, in orde to bring the concept of metamaterial into the experiment takes a long time, which is preceded by Pendry et al. in 1996 related to research of wire with a negative permittivity.[4] Then followed in 1999 regarding the material with negative permeability.[5] A year later, Pendry conducts experiments on negative refraction index material that could be used to manufacture the perfect lens.[6] In the same year, Smith et al.[7] doing research for verification if possible negative value electric permittivity and magnetic permeability simultaneously to a material under test. Some of these studies become the basis for the development of metamaterial studies by many researchers today.

The metamaterial is a unique artificial composite material with a negative refraction index value, which is not

available in nature.[1] Characteristics of metamaterial itself is a material that has a negative refractive index value in a certain wavelength range. The value of refraction index of a material is determined by the permittivity and permeability. Both of these is the basic characteristics that affect how the material's response to the presence of electromagnetic waves. [3] Based on the value of permittivity ε and permeability μ , material is classified into four categories : right-handed materials (RHM) $(\varepsilon > 0 \& \mu > 0)$, electric plasma $(\varepsilon < 0 \& \mu > 0)$, left-handed materials (LHM) $(\varepsilon < 0 \& \mu < 0)$ and magnetic plasma $(\varepsilon > 0 \& \mu < 0)$.

Analysis of the propagation of electromagnetic waves to a material mostly done through modeling. From modeling obtained can be used to determine how the material's response to the presence of electromagnetic waves. Wang et al. in 2008 have done research related to modeling of metamaterial wave propagation using finite difference time domain (FDTD) method. FDTD is a numerical method for computing the electromagnetic wave that has been widely used for modeling the propagation of electromagnetic waves. [8-9]

A theoretical approach is made to reduce the equation of electromagnetic waves in metamaterial into the second orde differential equation, like the Schrodinger equation or Dirac equation and then obtain the solution for distribution (eigenfunctions) and energy (eigenvalue). Analytical solution of NRM with graded refractive index interface between LHM and RHM has been given by Dalarsson and Tassin [10] for the case in which its index profile is a hyperbolic tangent function. The function of hyperbolic tangent is the most frequently used gradation profile. [11-12]. Recently, Husein et al. give the approximate solution of wave propagation in transverse magnetic mode through a graded interface positive-negative for the case the hyperbolic cosecant and cotangent functions using AIM. [2-13] Various methods to solve of the second orde differential equation has been used by many researchers, such methods Nikiforov-Uvarov [14-15], the method Supersymmetry Quantum Mechanics (SUSY QM) [16], the method of polynomial Romanovski[17] and Asymptotic Iteration Methods (AIM) [18-20].

In this paper, the wave propagation in transverse electric mode in a graded interface between left-handed and righthanded material will be investigated using AIM. The spatial variation in permittivity and permeability are described by a hyperbolic sinus plus hyperbolic tangent functions. The paper is organized as follows. The basic theory is presented in section 2. The results and discussion are presented in section 3 and a conclusion in section 4.

2. Basic Theory

2.1. The fields equation

The fields equation from Maxwell equation for dielectric, are given as

$$-\nabla \times \stackrel{1}{E} = i\omega\mu(\omega, y)\stackrel{1}{H}, \quad \nabla \times \stackrel{1}{H} = i\omega\varepsilon(\omega, y)\stackrel{1}{E} (1)$$

where $\mu(\omega, v)$ and $\varepsilon(\omega, v)$ are frequency and spatial dependent magnetic permeability and dielectric permittivity, respectively. [2,13] To simplify the calculations, we set the yz plane as the plane of ray incident, so (d/dx) = 0. In addition, the optical characteristics of $\varepsilon(y)$ and $\mu(y)$ are considered as functions of material thickness, y. Using Maxwell equation and considering the geometry of the material, we obtain a second orde differential equation of electromagnetic waves in inhomogeneous materials on mode transverse electric(TE). Actually, in two-dimensional study, there are two independent modes i.e. TE mode, the group of fields: $\left\{H_x,H_y,E_z\right\}$, $\left\{H_y,H_z,E_x\right\}$, $\left\{H_z,H_x,E_y\right\}$ and transverse magnetic (TM) mode, the group of fields: $\left\{E_x, E_y, H_z\right\}$, $\left\{E_y, E_z, H_x\right\}$, $\left\{E_z, E_x, H_y\right\}$.In principle, these two modes (TE and TM) are similar to each other, so we only need to do one calculation, e.g. the TE mode. By using the plane wave solution which is harmonic in time, is given

$$E_x = E(y, z)e^{i\omega t} \tag{2}$$

For the group fields in TE mode which we use i.e. $\{H_y, H_z, E_x\}$. From equation (1) and equation (2) we obtain

$$\frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} - \frac{\mu'}{\mu} \frac{\partial E_x}{\partial y} + \omega^2 \mu \varepsilon E_x = 0 \quad (3)$$

and by using $E_x = E(y, z) = E(y)E(z)$ in equation (3), we obtain

$$\frac{\partial^2 E(z)}{\partial z^2} + k_z^2 E(z) = 0 \tag{4}$$

$$\frac{\partial^2 E(y)}{\partial y^2} - \frac{\mu'}{\mu} \frac{\partial E(y)}{\partial y} + \left(\omega^2 \mu \varepsilon - k_z^2\right) E(y) = 0 \quad (5)$$

where k_z is the wave vector in z direction. Equation (4) is the simple differential equation which it has the solution,

$$E(z) = e^{\pm ik_z z} \tag{6}$$

while, for equation (5) with the graded interface permittivity and permeability is solved using AIM

2.2. Asymptotic Iteration Method

Asymptotic Iteration Method (AIM) is used to solve second orde differential equation in terms:

$$f_{n}''(l) - \lambda_{0}(l)f_{n}'(l) - s_{o}(l)f_{n}(l) = 0$$
 (7)

where $\lambda_0(l) \neq 0$ and $s_o(l)$ are coefficients of the differential equation and are well-defined functions as well as sufficiently differentiable. The solution of equation (7) can be obtained by using iteration of λ_a and s_a :

$$\lambda_{q}(l) = \lambda_{q-1} + \lambda_{q-1}\lambda_{0} + s_{q-1},$$

$$s_{q}(l) = s_{q-1} + \lambda_{q-1}s_{0}; q = 1, 2, 3, ...$$
(8)

Eigenvalues can be obtained using equation (9) below, [18-20]

$$\lambda_{q}(l)s_{q-1}(l) - \lambda_{q-1}(l)s_{q}(l) = 0 = \Delta_{q}, \tag{9}$$

For the eigenfunctions, following in [21] we use the Hermite differential equation. Hermite differential equation is given as.

$$f''(l) - 2lf'(l) + 2jf(l) = 0$$
 (10)

Equation (10) has the general solution in polynomial Hermite.

$$f_j(l) = (-1)^j e^{l^2} \frac{d^j}{ds^j} (e^{-l^2}); j = 0, 1, 2, 3...$$
 (11)

From equation (11) we can get some the first polynomial Hermite functions are given as

$$f_0(l) = 1, \quad f_1(l) = 2l$$

$$f_2(l) = 4l^2 - 2, \quad f_3(l) = 8l^3 - 12l,$$

$$f_4(l) = 16l^4 - 48l^2 + 12,$$
(12)

$$f_5(l) = 32l^5 - 160l^3 + 120l$$

Equation (12) are polynomial Hermite for j=0,1,2,3,4 and for j=5.

3. Discussion

In this research, we consider an inhomogeneous medium for which the permittivity and permeability vary according to a hyperbolic sinus function and a hyperbolic tangent function.[10] The permittivity and permeability are expressed as

$$\mu(\omega, y) = -\mu_0 \mu_r \left(\sinh(\rho y) + \tanh(\rho y) \right) (13)$$

$$\varepsilon(\omega, y) = -\varepsilon_0 \varepsilon_r \left(\sinh(\rho y) + \tanh(\rho y) \right) (14)$$

with ρ is abruptness's parameter describing the transition of wave propagation from RHM to LHM.

In below are the graphs of permeability (Fig. 1.a) and permittivity (Fig.1.b) vary according to the material thickness y where the interface between RHM and LHM is located at y = 0.

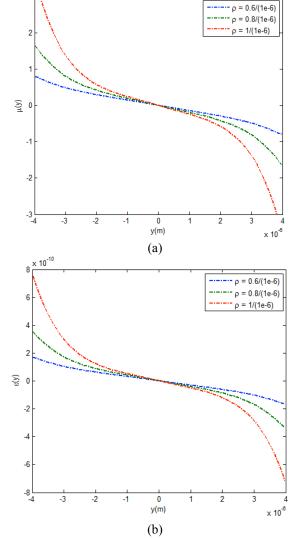


Figure 1: (a) permeability in hyperbolic function (b) permittivity in hyperbolic function

By inserting equations (13-14) into equation (5) we obtain the electromagnetic wave equation passing through the LHM media as follows

$$\frac{\partial^{2} E(y)}{\partial y^{2}} - \frac{\left(\rho\left(\cosh\left(\rho y\right) + \operatorname{sech}^{2}\left(\rho y\right)\right)\right)}{\left(\left(\sinh\left(\rho y\right) + \tanh\left(\rho y\right)\right)\right)} \frac{\partial E(y)}{\partial y} + (15)$$

$$\left(\omega^{2} \mu_{0} \mu_{r} \varepsilon_{0} \varepsilon_{r} \left(\sinh\left(\rho y\right) + \tanh\left(\rho y\right)\right)^{2} - k_{z}^{2}\right) E(y) = 0$$

by using the definition that

$$c = \left(1/\sqrt{\varepsilon_0 \mu_0}\right); \quad \mu_r \varepsilon_r = n_\omega^2; \quad k_0 = \left(\omega/c\right) \; ,$$

where c is light velocity in vacuum, n_{ω} is the refraction index and k_0 is wave vector in vacuum in equation (15), so equation (15) can be rewritten,

$$\frac{\partial^2 E(y)}{\partial y^2} - \frac{\rho \left(\cosh(\rho y) + \operatorname{sech}^2(\rho y)\right)}{\left(\sinh(\rho y) + \tanh(\rho y)\right)} \frac{\partial E(y)}{\partial y} + (16)$$
$$\left(k_0^2 n_\omega^2 \left(\sinh(\rho y) + \tanh(\rho y)\right)^2 - k_z^2\right) E(y) = 0$$

If we let $u = \rho y$, equation (16) can be rewritten as

$$\frac{d^2 E(y)}{du^2} - \frac{\left(\cosh\left(u\right) + \operatorname{sech}^2\left(u\right)\right)}{\left(\sinh\left(u\right) + \tanh\left(u\right)\right)} \frac{dE(y)}{du} + \frac{1}{\rho^2} \left(k_0^2 n_\omega^2 \left(\frac{\sinh\left(u\right) + \sum_{z=1}^{\infty} \left(\sinh\left(u\right) + \sum_{z=1}^{\infty$$

Equation (17) was approached for y is close to zero using the Maclaurin series, so we obtained

$$\cosh(u) + \operatorname{sech}^{2}(u) \approx 2;$$

$$\sinh(u) + \tanh(u) \approx 2u$$
(18)

which cause the second rate in equation (17) become

$$\frac{\left(\cosh\left(u\right) + \operatorname{sech}^{2}\left(u\right)\right)}{\left(\sinh\left(u\right) + \tanh\left(u\right)\right)} \approx \frac{1}{u}$$
(19)

However, equation (19) is not defined at u = 0, so it needs to make an assumption. If α is the value of u that close to zero then using equation (19) can be obtained intervals $-\alpha \le u \le \alpha$. By using this interval, equation (19) is considered to have the form of a linear function connecting the points $(-\alpha, -1/\alpha)$ and $(\alpha, 1/\alpha)$, ie

$$g(u) = \frac{1}{\alpha^2} u \tag{20}$$

so equation (17) was reduced become

$$\frac{d^{2}E(y)}{du^{2}} - \frac{u}{\alpha^{2}} \frac{dE(y)}{du} + \frac{1}{\rho^{2}} \left(4k_{0}^{2}n_{\omega}^{2}u^{2} - k_{z}^{2}\right)E(y) = 0$$
(21)

By letting $s = \left(4k_0^2 n_\omega^2/\rho^2\right)^{1/4} u$ into equation (21), we obtained

$$\frac{d^2 E(y)}{ds^2} - \frac{s}{\alpha^2} \frac{\rho}{2k_0 n_\omega} \frac{dE(y)}{ds} +$$

$$\left(s^2 - \frac{k_z^2}{2k_0 n_\omega \rho}\right) E(y) = 0$$
(22)

Equation (22) is AIM-type differential equation, like in equation (7) with

$$\lambda_0 = \frac{s}{\alpha^2} \frac{\rho}{2k_0 n_w}; s_0 = \frac{k_z^2}{2k_0 n_w \rho} - s^2$$
 (23)

for s is close to zero, we can approach equation (23) using Maclaurin series, given as

$$\lambda_{0}(s) = \lambda_{0}(0) + \lambda_{0}'(0)s + \lambda_{0}''(0)\frac{s^{2}}{2!} + \dots$$

$$= 0 + \frac{s}{\alpha^{2}}\frac{\rho}{2k_{0}n_{\omega}} + 0 + \dots \approx \frac{s}{\alpha^{2}}\frac{\rho}{2k_{0}n_{\omega}}$$

$$(24)$$

$$s_{0}(s) = s_{0}(0) + s_{0}'(0)s + s_{0}''(0)\frac{s^{2}}{2!} + \dots$$

$$= \frac{k_{z}^{2}}{2k_{0}n_{\omega}\rho} + 0 + \dots \approx \frac{k_{z}^{2}}{2k_{0}n_{\omega}\rho}$$

in equation (22), so we obtained

$$\frac{d^{2}E(y)}{ds^{2}} - \frac{s}{\alpha^{2}} \frac{\rho}{2k_{0}n_{\omega}} \frac{dE(y)}{ds} - \frac{k_{z}^{2}}{2k_{0}n_{\omega}\rho} E(y) = 0$$
 (25)

By letting $w = s \sqrt{\frac{\rho}{4k_0 n_\omega \alpha^2}}$ in equation (25), we obtained

$$\frac{d^{2}E(y)}{dw^{2}} - 2w\frac{dE(y)}{dw} - \frac{2k_{z}^{2}\alpha^{2}}{\rho^{2}}E(y) = 0$$
 (26)

Equation (26) is AIM-type differential equation, and by using AIM, from equation (9) we obtained,

$$\Delta_{j} = \prod_{j=0}^{m} \left[-\frac{2k_{z}^{2}\alpha^{2}}{\rho^{2}} - 2j \right]; m = 1, 2, 3...$$
 (27)

according to this condition, the eigenvalues can be obtained as follows

$$-\frac{k_z^2 \alpha^2}{\rho^2} = j \to k_z = i \frac{\rho}{\alpha} \sqrt{j}; j = 0, 1, 2, 3, \dots$$
 (28)

so equation (26) can be rewritten as

$$\frac{d^{2}E(y)}{dw^{2}} - 2w\frac{dE(y)}{dw} + 2jE(y) = 0$$
 (29)

which known as Hermite differential equation. The approximate solution of equation (29) is

$$H_{j}(y) = (-1)^{j} e^{w^{2}} \frac{d^{j}}{dw^{j}} \left(e^{-w^{2}}\right)$$
 (30)

where

$$w = \frac{\rho}{\alpha\sqrt{2}}y, -\frac{1}{\sqrt{2}} \le w \le \frac{1}{\sqrt{2}}, \alpha = 1$$
 (31)

From equation (28), we can obtain the wave vector k_{z_j} . Wave vector is dependent on the abruptness's parameter, the material thickness and the level j. In Fig. 2 is the 2-dimension graphs between $k_{z_j}^2$ versus p in any level j. The amplitude of wave vector is increase with increasing of abruptness's parameter in the same level, and with the higher level, the amplitude of wave vector also increase, is like shown in Fig. 2.

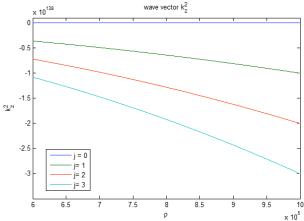


Figure 2: The graph of wave vector $k_{z_j}^2$ with variation j and p

Then for the fields distribution, we investigated from the total eigenfunctions using equations (6,12,30-31) for j=1,2,3 and j=4, and we take the positive sign for E(z), we obtain:

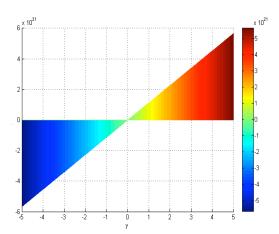
for
$$j=1$$
, $E_{x_1} = E_1(y,z) = e^{ik_z z}$ (32)

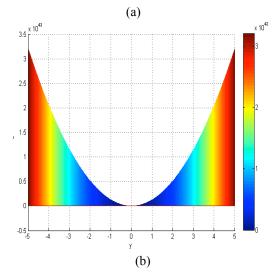
for
$$j=2$$
, $E_{x_2} = E_2(y,z) = \left(\frac{2\rho^2}{\alpha^2}y^2 - 2\right)e^{ik_z z}$ (33)

for
$$j=3$$
, $E_{x_3} = E_3(y,z) = \left(\frac{4\rho^3}{\alpha^2 \sqrt{2}} y^3 - \frac{12\rho}{\alpha \sqrt{2}} y\right) e^{ik_z z}$ (34)

for
$$j=4$$
, $E_{x_4} = E_4(y,z) = \begin{pmatrix} \frac{4\rho^4}{\alpha^4} y^4 - \\ \frac{24\rho^2}{\alpha^2} y^2 + 12 \end{pmatrix} e^{ik_z z}$ (35)

according to equations (32-35), we obtained the electric fields distribution which is shown in Fig. 3. Fig. 3 is the graphs of electric fields distribution in various level. Fig. 3.a is the first level j=1 from equation (32), Fig.3.b from equation (33) for the second level j=2, Fig. 3.c is the third level from equation (34), and Fig.3.d is the fourth level from equation (35). The graphs show that in various the material thickness y, the electric field has the different distribution. The amplitude of electric fields are increase with higher level. Wave propagation of electric field E(y,z) which is expressed in Fig. 3 give explanation that in a graded interface between left-handed and right-handed material using hyperbolic sinus plus tangent function has the symmetry propagation with y=0 is the interface point.





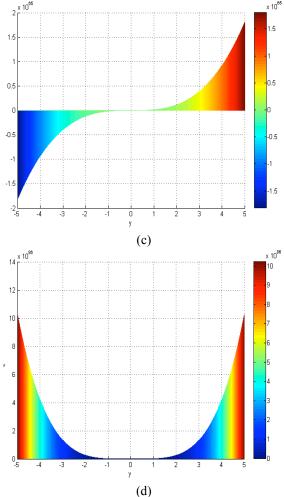


Figure 3: Distribution of the electric fields (a) j=1, (b) j=2 (c) j=3 and (d) j=4

4. Conclusions

Asymptotic iteration method was used to investigated the wave propagation for transverse electric (TE) mode in a graded interface between left-handed and right-handed material. By using hyperbolic sinus function plus hyperbolictangent function for negative permittivity and negative permeability, we obtained the graded graphs of permittivity and permeability as a function of material thickness. The wave vector was obtained using AIM, while the eigenfunctions were expressed using polynomial Hermite. The distribution of the electric field was shown in 2-dimension graphs, which has the symmetry propagation with y=0 is the interface point. The amplitude of wave vector is increase with the higher level. The results of this research used the approximation solution.

Acknowledgements

This research was partly supported by Higher Education Project Grant with contract no. 632/UN27.21/LT/2016.

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