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Research on Mathematical Modeling and Control Strategy of Urban Traffic Flow Congestion Evolution with Time Delay Stochastic Differential Equations

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ABSTRACT Urban road congestion is not simply caused by traffic demand exceeding capacity, but is a dynamic evolutionary result of the coupling of vehicle arrival randomness, driving response lag, signal control delay, path guidance feedback, accident disturbance, and road network bottlenecks. The traditional deterministic traffic flow model can describe the average relationship between density, speed, and flow, but it lacks sufficient explanation for phenomena such as peak period fluctuations amplification, sudden congestion transfer, and control lag failure. This article uses stochastic differential equations with time delay as a tool to construct an urban traffic congestion evolution model based on the absorption of macroscopic traffic flow theory, stochastic dynamical systems, and feedback control ideas. The model takes road density, average speed, queue length, and congestion intensity as state variables, introduces demand disturbance, capacity disturbance, and perceived noise, and characterizes the delay of driving response, detection and acquisition, signal execution, and information release through discrete and distributed time delays. Further research provides methods for congestion equilibrium state, mean square stability, threshold discrimination, and control strategy design, and proposes a governance framework that combines signal coordination, speed induction, demand regulation, path diversion, and model predictive control. The results show that the combined effect of time delay and noise will significantly reduce the system stability margin. Only by incorporating the control advance, feedback period and disturbance intensity into the unified model can the reliability of urban traffic congestion governance be improved.

INDEX TERMS Stochastic differential equations with time delay; Urban traffic flow; Congestion evolution; control strategy

I. INTRODUCTION

A. RESEARCH BACKGROUND

Urban traffic flow has typical characteristics: changes in road density are not instantaneous feedback to all vehicles, and control strategies do not immediately change the behavior of all travelers. Any control action must go through a chain of recognition, decision-making, execution, and response. Therefore, stochastic differential equations with time delay are more suitable for characterizing delayed feedback and uncertain disturbances in congestion evolution than ordinary deterministic models [1-2].

B. RESEARCH SIGNIFICANCE

From a theoretical perspective, this article introduces time-delay stochastic differential equations into the study of urban traffic congestion evolution, which helps to compensate for the insufficient explanation of stochastic fluctuations and delayed feedback in deterministic models. Traditional congestion analysis is often based on average traffic flow and average speed, which are suitable for describing stable or quasi stable states, but it is difficult to answer why congestion sometimes forms rapidly and sometimes naturally dissipates at the same demand level; Why are certain control strategies effective in simulation but prone to over regulation or repeated oscillations in actual operation [3-4]. After introducing random terms, the model can convert vehicle arrivals, lane changes, weather, events, and detection errors into analyzable disturbances; After introducing the time delay term, the model can incorporate driver response, control execution, and information induced lag into the same power system. In this way, congestion research has shifted from simply analyzing the relationship between capacity and demand to analyzing the stable boundary determined by disturbances, feedback, and time delays [5-6].

From a practical perspective, urban transportation governance is increasingly reliant on data-driven and real-time control. The traffic management department collects a large amount of data through video, geomagnetic, floating cars, navigation platforms, and signal machines, hoping to achieve green wave coordination, variable speed guidance, ramp control, bus priority, and regional diversion. However, the more real-time the data, the more stable the control does not necessarily mean. If the control cycle is too short and the detection error is large, the signal timing may change frequently, triggering new queues; If the path guidance is released too slowly, vehicles may concentrate on turning to alternative roads, causing congestion migration; If the speed control ignores the driving response time delay, it may cause the deceleration wave to amplify upstream. This article incorporates time delay, noise, and control gain into the model, providing mathematical basis for control cycle setting, strategy combination, and risk assessment.

C. RESEARCH APPROACH

The research approach of this article includes four levels. Firstly, based on the basic traffic flow diagram and conservation relationships, define core variables such as urban road density, average speed, traffic flow, queue length, and congestion intensity, and explain their interactions. Secondly, construct a stochastic differential equation model with time delay, incorporating traffic demand disturbances, capacity fluctuations, driving behavior differences, and detection errors into the diffusion term, and incorporating driving responses, signal execution, and information induction into the time delay term. Thirdly, analyze the evolution of congestion from the perspectives of equilibrium state, mean square stability, congestion threshold, and noise sensitivity. Fourthly, around the design of control strategies, propose a comprehensive framework for signal coordination, speed induction, demand regulation, path diversion, and model predictive control, and discuss implementation guarantees. For ease of modeling in the following text, the main variables and symbols are shown in Table 1.

TABLE 1. Model Variables and Symbols

Type	Variable	Symbol
State	Traffic density	$\rho(t)$
State	Mean speed	$v(t)$
State	Queue length	$q(t)$
State	Congestion intensity	$c(t)$
Input	Arrival demand	$d(t)$
Input	Signal control	$u_s(t)$
Input	Speed guidance	$u_v(t)$
Noise	Demand and capacity disturbance	$\sigma\, dW(t)$
Delay	Reaction and execution lag	τ

II. THEORETICAL BASIS AND MODEL ASSUMPTIONS

A. EVOLUTION MECHANISM OF URBAN TRAFFIC CONGESTION

The formation of traffic congestion first depends on the relationship between demand and capacity. When the release capacity of vehicles exceeds that of roads, intersections, or bottlenecks per unit time, the queue of vehicles increases, the speed decreases, the density increases, and the system enters a congested state. However, urban traffic congestion does not always increase linearly due to the fluctuating propagation and feedback amplification characteristics of traffic flow. A short-term overflow at a certain intersection will occupy the exit of the upstream intersection, hinder the release of lateral traffic, and further cause regional congestion. On a microscopic level, the driver's braking, following, and lane changing behaviors will form speed waves; Macroscopically, density waves propagate upstream along the road, transforming local disturbances into continuous queues. The cellular automaton model proposed by Nagel and Schreckenberg indicates that individual random deceleration is sufficient to form spontaneous congestion under high-density conditions [7]. This provides a behavioral basis for introducing random perturbations in continuous models [8-9].

B. APPLICABILITY OF STOCHASTIC DIFFERENTIAL EQUATIONS WITH TIME DELAY

Time delayed stochastic differential equations are suitable for describing systems where the current state is influenced by both historical states and random disturbances. The general form can be written as:

$$\begin{aligned} dX(t) &= f(X(t), X(t-\tau), u(t))\, dt \\ &\quad + g(X(t), X(t-\tau))\, dW(t) \end{aligned}$$

Where $X(t)$ represents the current traffic state, $X(t-\tau)$ represents the lagged state, u (t) represents the control input, and $W(t)$ is the Brownian motion or equivalent random disturbance process. For traffic flow, $X(t-\tau)$ can represent the driver's response based on the speed difference several seconds ago, or it can represent the signal control system adjusting the phase based on the data from the previous detection cycle. The theory of functional differential equations states that the stability of time-delay systems is not only determined by current feedback, but also by historical trajectories and lag lengths [10]; Random time-delay systems are further affected by noise intensity and diffusion structure [11].

C. MODEL ASSUMPTIONS

In order to maintain the interpretability and usability of the model for control design, the following basic assumptions are made in this paper. Firstly, the research object is urban expressways or main road corridors, which can be divided into several continuous sections, with the average density, average speed, and queue length representing the status within each section. Secondly, the demand for vehicle arrival is composed of deterministic benchmark demand and random disturbances, with disturbance intensity varying over time periods and event states. Thirdly, there is an average delay of

τ_d in the driver's response to the state and traffic information ahead, a control delay of τ_2 in the signal system from detection to execution, and an induction delay of τ_i in the navigation guidance from issuance to path formation. Fourthly, road capacity is affected by weather, events, and lane occupancy, resulting in random capacity fluctuations. Fifth, the control strategy does not directly eliminate demand, but affects state evolution by changing the signal green signal ratio, target speed, diversion ratio, and demand entry rate. The meanings of model parameters and data sources are shown in Table 2 below.

TABLE 2. Parameter Interpretation and Data Sources

Parameter	Meaning	Unit	Source	Range	Role
α	Density recovery coefficient	1/min	Detector calibration	0.05–0.30	Return to free flow
β	Speed-density sensitivity	km/h	Floating car data	0.10–0.80	Speed decline
γ	Queue spillback coefficient	veh/min	Signal records	0.05–0.50	Queue growth
σ	Random disturbance intensity	mixed	Historical residuals	0.01–0.20	Noise scale
τ	Average delay length	s	Control log	5–90	Feedback lag

III. CONSTRUCTION OF STOCHASTIC DIFFERENTIAL EQUATION MODELS WITH TIME DELAY

A. MODEL EXPRESSION

This article presents a simplified model for the evolution of urban traffic congestion:

$$dX(t) = F(X(t), X(t-\tau_d), U(t-\tau_2), D(t-\tau_i))\ dt + G(X(t), \theta)\ dW(t)$$

Among them, F represents the deterministic drift term, reflecting demand entry, flow release, speed adjustment, queue growth, and control effects; G represents the diffusion term, reflecting the strength of random disturbances; U represents control input, including signal green ratio adjustment, speed induction, path diversion, and demand adjustment; D represents external demand. This model is not intended to replace all traffic simulation platforms, but to provide a mathematical framework for stability analysis and control strategy comparison [12]. Different cities and road types can be adjusted in the specific functional forms of F and G.

The model also needs to introduce congestion state classification. Traffic states can be classified into six categories based on the size and trend of $c(t)$: free flow, critical flow, local congestion, diffusion congestion, recovery period, and secondary congestion risk. In the free flow state, the density is low and the velocity is high, and random disturbances will rapidly decay; Under critical flow conditions, the system approaches the capacity boundary, and small disturbances may be amplified; Queuing formed under local congestion but did not overflow; Upstream road sections are blocked under diffusion congestion conditions;

During the recovery period, the traffic is lower than the capacity but the speed has not been fully restored; The risk of secondary congestion is manifested as the rapid release of control measures leading to a re increase in queues. The discriminative features and modeling meanings of each state are shown in Table 3 below.

TABLE 3. Congestion Evolution States

State	Density Level	Speed Pattern	Queue Feature	Main Driver	Model Implication
Free flow	Low	Stable high speed	No queue	Normal demand	Noise decays quickly
Critical flow	Near threshold	Speed variance rises	Short queue	Demand fluctuation	Small noise may amplify
Local congestion	High on one link	Speed drops	Queue forms	Bottleneck delay	Control should act early
Spreading congestion	High on adjacent links	Stop-and-go	Spillback	Network coupling	Regional coordination needed
Recovery	Declining	Slow rebound	Queue dissipates	Capacity restored	Avoid premature release

IV. ANALYSIS OF CONGESTION EVOLUTION AND STABILITY DISCRIMINATION

A. EQUILIBRIUM STATE AND CONGESTION THRESHOLD

Kerner's research on the three-phase theory of traffic flow emphasizes that congestion is not a single form, and there may be transitions between states such as dense synchronous flow and wide motion congestion [13]. This inspires us not to only set a congestion threshold in mathematical modeling, but to consider state transition boundaries.

B. TIME DELAY EFFECTS AND SENSITIVITY

The impact of time delay on congestion evolution is mainly reflected in three aspects. Firstly, time delay reduces feedback accuracy. The control system makes decisions based on past states, and if the traffic conditions change rapidly, the actual state at the time of the decision may have been different. Secondly, time delay can cause oscillations. The signal system increases the green light after seeing an increase in upstream queues, but when vehicles arrive downstream, it causes downstream congestion. Subsequently, the system adjusts in reverse, forming periodic fluctuations. Thirdly, time delay amplifies random disturbances. The noise may have dissipated in a short period of time, but lag control will treat the disappeared disturbance as the current problem and continue to handle it, causing the system to deviate from equilibrium [13-14].

V. CONTROL STRATEGY DESIGN

A. SIGNAL SYNERGY AND SPEED INDUCTION

Signal collaborative control is the most common means of urban traffic management. Traditional green wave coordination mainly relies on the average speed in the main direction and the distance between intersections to set the phase difference. However, in the stochastic time-delay model, signal control should also pay attention to the variance of queue length, the direction of velocity wave propagation, and downstream acceptance capacity. If the downstream intersection is close to saturation, continuing to increase the upstream green light may lead to blockage and overflow; If the upstream queue is long and there is still capacity downstream, it should be released in advance. The control input $u_2(t)$ can be represented as a combination of green signal ratio, cycle length, and phase difference, which has an effect on the execution delay τ_2. To avoid oscillation, signal control should not overreact to single detection anomalies, but should be adjusted smoothly based on short-term predictions and confidence intervals.

Speed induction is suitable for continuous flow scenarios such as urban expressways, tunnels, bridges, and elevated roads. Its goal is not simply to reduce speed, but to suppress disturbance amplification by reducing speed differences and lane changing conflicts. If the model predicts that the road ahead is about to enter a critical state, a lower but stable target speed can be released upstream to make the flow rate of vehicles reaching the bottleneck more uniform. The effect of speed induction is also affected by the delay in driving response. If the release speed is too late and the vehicle has entered the congestion wave range, the induction effect is limited; If the target speed changes too quickly, the driver's compliance rate decreases and instead increases speed dispersion. The combination of signal synergy and speed induction is shown in Table 4 below.

TABLE 4. Control Strategy Matrix

Strategy	Control Variable	Main Delay
Signal coordination	Green ratio and offset	Detection and execution lag
Variable speed guidance	Target speed	Driver reaction lag
Ramp or access control	Entry rate	Queue acceptance lag
Route diversion	Diversion ratio	Information response lag
Bus priority	Priority phase	Schedule and stop delay
Parking guidance	Cruising demand	Information update lag
Incident management	Capacity recovery	Detection and clearance lag

B. DEMAND REGULATION AND PATH DIVERSION

Demand regulation emphasizes reducing peak entry rates from the source. Urban traffic congestion is often triggered by demand pulses in a short period of time. If demand can be dispersed through off peak travel, parking prices, congestion warnings, bus connections, and reservation services, the probability of the system crossing critical thresholds can be reduced. In the model, demand regulation can be manifested as a decrease in the mean or variance of $d(t)$. Compared with signal control, demand regulation has a longer time delay, but once stable behavior is formed, the effect is more long-lasting. For obvious tidal demand sources such as schools, hospitals, commercial districts, exhibition centers, and large residential areas, demand pulses can be estimated in advance based on historical data, and guidance can be initiated before congestion forms.

C. ROBUST CONTROL AND MODEL PREDICTION

Model predictive control is suitable for handling multi input, multi constraint, and rolling optimization problems. Hegyi et al. studied the model predictive control of ramp control and variable speed limit coordination, and demonstrated that coordinating multiple control measures can improve the operational efficiency of highways [15]. In urban transportation, model predictive control can roll estimate the density, speed, and queue length for several minutes in the future every cycle, and find a combination strategy between signal, speed, diversion, and demand regulation. Due to the existence of time delay in the system, the prediction window must cover the main feedback delay; Due to the presence of random disturbances, the objective function should include expected congestion intensity and risk penalty terms. The evaluation of control effectiveness should not only focus on average speed, but also on the probability of queue overflow, reliability of travel time, and risk of secondary congestion.

VI. SIMULATION VERIFICATION AND IMPLEMENTATION MECHANISM

A. SIMULATION SCENE DESIGN

To test the model and control strategy, a typical urban corridor simulation scenario can be set up. The simulation object consists of a continuous main road, several signalized intersections, branch merging, and downstream bottlenecks. The basic demand is given in three stages: morning peak rise, platform period, and decline, and is superimposed with random arrival disturbances. The capacity disturbance is set to four categories: mild weather, temporary parking, accident occupation, and detection error. Control strategies include uncontrolled, single point signal control, signal coordination, speed induction, path diversion, and comprehensive model predictive control. Comparative indicators include average delay, maximum queue length, congestion duration, speed variance, secondary congestion frequency, and control smoothness. The simulation scenario design is shown in Table 5 below.

TABLE 5. Simulation Scenario Design

Scenario	Disturbance	Control	Expected Use
Baseline	Normal demand noise	Fixed signal	Reference comparison
Incident	Capacity reduction	Coordinated control	Spillback prevention

During the simulation process, parameter estimation can be based on historical detection data and floating vehicle data. Firstly, fit the critical density and free flow velocity using a basic graph, and then estimate the random disturbance intensity using residuals; Secondly, use signal control logs to estimate execution delay, and use speed changes and queuing response to estimate driving response delay; Finally, calibrate the capacity reduction and recovery process using historical congestion events. The high-speed traffic state estimation method based on extended Kalman filter proposed by Wang and Papageorgiou shows that the traffic state can be more stably estimated through model and observation fusion [16]. In urban road networks, Kalman filtering, particle filtering, or Bayesian update methods can also be used to perform rolling correction on density, speed, and queuing status.

B. EFFECT EVALUATION

The evaluation of effectiveness should simultaneously focus on efficiency, stability, and feasibility. Efficiency indicators include average travel time, intersection delay, traffic volume, and queue length; Stability indicators include speed variance, congestion intensity variance, number of state threshold crossings, and disturbance attenuation time; The feasibility indicators include the number of control actions, phase change amplitude, driver compliance rate, and impact on public transportation and slow traffic. Research on speed data assimilation by Work et al. has shown that using mobile data to improve traffic state estimation can enhance the explanatory power of models for actual traffic evolution [17-18]. Therefore, the evaluation system should also include data quality to avoid mistaking detection errors for control effectiveness.

C. LANDING GUARANTEE AND RISK CONTROL

In terms of risk control, it is necessary to focus on four types of issues. Firstly, there is a risk of data bias. If the floating car sample is concentrated on a specific vehicle type, the model may misjudge the overall speed. Secondly, control the risk of oscillation. Short cycles and excessive gain can cause frequent signal switching. Thirdly, congestion transfer risk. The diversion strategy may transfer the pressure from the main road to residential roads. Fourth, fairness risk. Excessive protection of traffic in one direction may harm the interests of buses, pedestrians, and residents on the side roads. The implementation checklist and risk control points are shown in Table 6 below.

TABLE 6. Implementation Risks and Checks

Risk	Manifestation	Control Check
Data bias	Sample speed differs from full traffic	Multi-source data validation
Control oscillation	Frequent phase or speed changes	Gain limit and smoothing rule
Congestion transfer	Main road improves while side road worsens	Network-wide objective function
Delay underestimation	Control arrives after state changes	Delay calibration and prediction window
Low compliance	Drivers ignore guidance	Adaptive compliance parameter
Equity issue	Priority harms bus or pedestrian service	Public service constraint
System failure	Detector or platform outage	Fallback plan and manual override

VII. CONCLUSION

This article focuses on the stochastic disturbance and feedback lag problems in the evolution of urban traffic flow congestion, and constructs a framework for analyzing stochastic differential equations with time delay. Based on this, it discusses congestion state discrimination, stability analysis, and control strategy design. Research suggests that urban congestion is not a simple static result of demand exceeding capacity, but a dynamic process that combines the randomness of vehicle arrivals, fluctuations in traffic capacity, delayed driving responses, delayed signal execution, and information induced feedback. Incorporating density, speed, queue length, and congestion intensity into the state vector, and introducing demand disturbance, capacity disturbance, and observation noise, can better explain phenomena such as sudden congestion during peak hours, amplification of speed waves, and secondary congestion.

In terms of control strategy, signal coordination, speed induction, demand regulation, path diversion, and model predictive control should be combined and not used in isolation. The longer the time delay and the stronger the noise, the more it is necessary for the control strategy to maintain advance, smoothness, and robustness. Future research can further combine measured data to estimate model parameters under different road network structures, driver compliance rates, and event disturbances, and explore the integration of stochastic control, reinforcement learning, and traffic digital twin platforms. The non-equilibrium traffic model proposed by Zhang emphasizes that speed dynamics cannot be simply equated with gas behavior [19], and traffic flow mechanics still need to respect driving behavior and road constraints; In recent years, textbooks and research on traffic flow mechanics have continuously emphasized the consistency between data calibration, model interpretation, and control implementation [20]. Therefore, stochastic differential equations with time delay are not simply mathematical forms, but important tools for connecting traffic mechanisms, data estimation, and control governance.

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