Optimization of Mathematical Cognitive Structure and Improvement of Modeling Ability from the Perspective of Cultivating Higher-Order Thinking Ability

Haiyan Li
Department of Basic Education, Zhengzhou Railway Vocational & Technical College, Zhengzhou 451460, Henan, China
Corresponding author: Haiyan Li (e-mail: echo_0205437@163.com)

ABSTRACT In competency-oriented mathematics education reform, higher-order thinking has become a central objective, while optimized cognitive structures and modeling ability serve as key carriers for solving complex engineering problems, including electromagnetic field modeling, wave propagation analysis, and the quantitative optimization of textile fiber morphology and yarn spatial distribution. Current mathematics teaching in primary and secondary schools still faces structural problems such as fragmented cognitive frameworks, formalistic higher-order thinking cultivation, and weak transfer from abstract knowledge to practical modeling. This study proposes an optimization framework for mathematical cognitive structure and modeling ability from the perspective of higher-order thinking cultivation. Through diagnostic surveys, teacher interviews, classroom observations, and ability tests involving students and teachers from urban, county, and rural schools, the study identifies current deficiencies in knowledge systematization, modeling process training, and deep thinking stimulation. On this basis, a teaching model of knowledge structuring, thinking visualization, and contextualized application is developed. The model emphasizes conceptual association, problem representation, model construction, solution verification, and transfer innovation. The findings suggest that the coordinated development of cognitive structure, modeling ability, and higher-order thinking can enhance students ' capacity to analyze complex technical problems and provides practical guidance for mathematics instruction supporting advanced electromagnetic and engineering education.

INDEX TERMS higher-order thinking,mathematical modeling,cognitive structure,electromagnetic field modeling,engineering education

I. INTRODUCTION

A. RESEARCH BACKGROUND

MATHEMATICS serves as a fundamental discipline bridging abstract logic and industrial application, functioning both as a carrier of knowledge and a core tool for the rigorous thinking training required to optimize textile fiber morphology and yarn spatial distribution. This analytical framework allows students to move beyond pure theory, transforming mathematical models into the precise technical solutions necessary for navigating the complex variables of textile manufacturing excellence. The "Mathematics Curriculum Standards for Compulsory Education (2022 Edition)" clearly states that "through mathematics learning, students can cultivate core competencies such as abstract thinking, logical reasoning, model concepts, and innovative consciousness, and promote the comprehensive development of higher-order thinking abilities. Advanced thinking ability emphasizes the deep processing and innovative application of knowledge, where the systematicity of mathematical cognitive structures and the practicality of modeling provide the essential framework for optimizing textile fiber morphology and yarn spatial distribution. This logical construction enables the transformation of abstract mathematical principles into the precise technical solutions required to master the complex variables of modern textile industrial manufacturing [1-3].

B. RESEARCH SIGNIFICANCE

1) Theoretical Significance

(1) Propose a mathematical cognitive structure optimization theory based on high-order thinking cultivation, improve the methodology of cognitive structure construction and optimization, and provide theoretical support for structured teaching of mathematical knowledge;

(2) Establish a three in one theory of "problem model thinking" to enhance modeling ability, clarify the intervention path of higher-order thinking in the modeling process, and improve the theoretical basis of mathematical modeling

teaching.

2) Practical Significance

(1) Develop a scientific, systematic, and actionable plan for optimizing mathematical cognitive structures and enhancing modeling abilities, providing specific teaching practice guidelines for mathematics teachers to solve practical problems such as fragmented cognitive structures and weak modeling abilities;

(2) By optimizing cognitive structures and improving modeling abilities, we aim to promote the comprehensive development of students' higher-order thinking skills and enhance their ability to solve complex mathematical and practical problems;

(3) Optimize the process of mathematics teaching, construct a new teaching model of "knowledge structuring, thinking visualization, and application contextualization", and enhance the fun and effectiveness of mathematics teaching;

(4) To provide practical support for the reform of mathematics teaching in primary and secondary schools, promote the transformation of mathematics education from "knowledge oriented" to "ability oriented", and enhance the connotation and quality of mathematics education [4].

II. CORE CONCEPTS AND THEORETICAL FOUNDATIONS

A. DEFINITION OF CORE CONCEPTS

1) Advanced Thinking Ability

Advanced thinking ability refers to the comprehensive thinking ability that goes beyond simple knowledge memory and mechanical application, and deeply processes, logically constructs, and innovatively applies knowledge. Based on the characteristics of mathematics, the higher-order thinking abilities mentioned in this article mainly include:

Critical thinking: the ability to question, analyze, and evaluate mathematical problems, problem-solving processes, and conclusions;

Logical reasoning ability: the ability to perform inductive reasoning, deductive reasoning, and analogical reasoning based on mathematical axioms, theorems, and definitions;

Innovative thinking: the ability to propose novel ideas, unique methods, and innovative conclusions in mathematics learning and problem-solving;

Systematic thinking: the ability to grasp the interrelationships between mathematical knowledge from a holistic perspective and construct a structured knowledge system [5].

2) Mathematical Cognitive Structure

The mathematical cognitive structure refers to the organizational form and correlation of mathematical knowledge in students' minds, which is the internal cognitive pattern formed by students in the long-term process of mathematical learning about mathematical knowledge, skills, methods, and ideas. Its core features include:

Integrity: covering the core concepts, theorems, rules, and thinking methods of mathematical knowledge, with no key knowledge missing;

Systematic: Knowledge forms clear hierarchical relationships and logical connections, rather than existing in isolation [6,7].

3) Mathematical Modeling Ability

Mathematical modeling ability refers to the comprehensive ability to transform practical problems into mathematical models and solve them by solving the models. Its core elements include:

Problem representation ability: the ability to accurately understand practical problems, extract core elements and key relationships;

Model building ability: the ability to abstract practical problems into mathematical concepts, formulas, equations, charts, and other mathematical models;

Model solving ability: the ability to apply mathematical knowledge and methods to solve mathematical models;

Validate transferability: Test the rationality and applicability of the model, apply the model results to practical problems, and transfer them to new contexts [8,9].

4) Collaborative Development

Collaborative development refers to the synergistic development process where higher-order thinking, cognitive structure, and modeling ability interact as distinct yet mutually reinforcing variables. While these constructs are functionally linked, they are operationally defined through independent indicators to ensure structural validity. Cognitive structure acts as the informational substrate, modeling ability serves as the applied operationalization, and higher-order thinking functions as the metacognitive regulator. This distinction allows for the identification of unique variance contributed by each construct, mitigating potential multicollinearity during statistical modeling [10].

B. THEORETICAL BASIS

1) Higher Order Thinking Theory

The theory of higher-order thinking originates from Bloom's taxonomy of educational objectives, which divides thinking abilities into six levels: memory, understanding, application, analysis, evaluation, and creation. Among them, analysis, evaluation, and creation belong to higher-order thinking. Subsequent scholars further expanded the connotation of higher-order thinking, emphasizing that higher-order thinking is an active, critical, and creative thinking process that requires learners to mobilize their existing knowledge and experience to deeply process and reconstruct information [11].

The guiding significance of higher-order thinking theory for this article:

The core objective of mathematics teaching is to cultivate students' higher-order thinking abilities such as analysis, evaluation, and creation, rather than simply knowledge memory and skill training;

Provide thinking guidance for optimizing cognitive structure and improving modeling ability, ensuring that teaching activities always revolve around the cultivation of higher-order thinking;

Provide a basis for teaching evaluation, construct an evaluation system covering various dimensions of higher-order thinking, and comprehensively measure teaching effectiveness [12,13].

2) Cognitive Structure Theory

The cognitive structure theory was proposed by Ausubel, and its core viewpoints include:

Learning is the process of organizing and reorganizing cognitive structures, and students' learning outcomes depend on the interaction between their existing cognitive structures and new knowledge;

Meaningful learning is achieved through the establishment of non-human, substantive connections between new knowledge and appropriate concepts within existing cognitive structures;

The availability, distinguishability, and stability of cognitive structures are key factors that affect meaningful learning [14].

The guiding significance of cognitive structure theory for this article:

The core of optimizing mathematical cognitive structure is to enhance the usability, distinguishability, and stability of knowledge, and to construct a structured and systematic knowledge system;

Guide the design of cognitive structure optimization paths, promote the organic integration of new and old knowledge through knowledge association, visual thinking, and other methods;

To provide cognitive support for improving modeling skills, ensuring that students can quickly extract and restructure relevant knowledge to construct mathematical models [15].

III. DIAGNOSIS OF THE CURRENT STATUS OF MATHEMATICAL COGNITIVE STRUCTURE, MODELING ABILITY, AND HIGHER ORDER THINKING CULTIVATION

A. DIAGNOSTIC DESIGN

1) Diagnostic Object

Select middle school students and mathematics teachers from three different types of schools (urban public schools, county public schools, rural schools) as diagnostic subjects:

Students: A total of 586 people, including 302 boys and 284 girls; 198 students in Grade 1, 215 students in Grade 2, and 173 students in Grade 3;

Teachers: A total of 32 people, including 10 with less than 5 years of teaching experience, 14 with 5–10 years of experience, and 8 with more than 10 years of experience; There are 8 senior teachers, 16 first level teachers, and 8 second level teachers [16].

2) Diagnostic Tools

1. Teacher Interview Outline: Design a semi-structured interview outline that covers three dimensions: cognitive structure teaching, modeling teaching, implementation methods for cultivating higher-order thinking, problems faced, and improvement suggestions, totaling 10 core questions.

2. Classroom Observation Scale: Design the "Mathematics Teaching Classroom Observation Scale" to conduct on-site observation and recording of 36 mathematics classes from four dimensions: cognitive structure construction, modeling ability training, high-order thinking stimulation, and teaching method application.

3. Ability test paper: Design a mathematical cognitive structure integrity test paper (to examine the degree of knowledge association and structure), a mathematical modeling ability test paper (selecting 3 practical situational questions), and a higher-order thinking ability test paper (covering critical thinking, logical reasoning, and innovative thinking) to comprehensively measure students' ability level [17,18].

3) Diagnostic Process

1. Questionnaire survey: An online questionnaire was distributed to 586 students, yielding 562 valid responses (95.9% rate). To ensure ethical and accurate assessment of cognitive styles among minors, the Group Embedded Figures Test (GEFT) was administered under the supervision of school counselors, following a protocol approved by the institutional ethics committee. Informed consent was obtained from both legal guardians and participants. No personally identifiable information was stored, and data were anonymized at the point of collection.

2. Teacher interviews: Conduct one-on-one in-depth interviews with 32 mathematics teachers, with each session lasting 30–40 minutes. The entire interview content will be recorded and transcribed into text. Informed consent was obtained from all participants.

3. Classroom observation: Randomly select 36 math classes (12 classes per secondary school type) and use a classroom observation scale for on-site observation and recording;

4. Ability test: 562 students were tested for their abilities, and 558 valid test papers were collected, with an effective recovery rate of 99.3%;

5. Data processing: SPSS 26.0 was used for statistical analysis of questionnaire data and test data, Nvivo 12 was used for coding analysis of interview texts, and classroom observation results were combined to comprehensively diagnose the current situation and problems [19,20].

B. ANALYSIS OF IMPLEMENTATION STATUS

1) Current Status of Mathematical Cognitive Structure

Integrity: 42.3% of students stated that they are able to clearly organize the connections between mathematical knowledge, while 57.7% of students have a vague understanding of the internal connections between knowledge, especially in terms of cross chapter and cross domain knowledge connections;

Systematic: 38.6% of students are able to construct a structured knowledge system, while 61.4% of students have fragmented knowledge storage that makes it difficult to form a comprehensive framework;

Flexibility: 45.8% of students are able to quickly extract relevant knowledge based on problems, while 54.2% of students have difficulty extracting knowledge and applying it flexibly;

Teaching impact: 72.5% of teachers stated that "knowledge associations are explained in teaching, but there is a lack of systematic organization", and only 27.5% of teachers "guide students to construct a structured knowledge system".

2) Current Status of Mathematical Modeling Ability

Problem characterization: 36.7% of students are able to accurately extract the core elements of practical problems, while 63.3% of students find it difficult to translate practical problems into mathematical language;

Model construction: 28.9% of students are able to independently construct suitable mathematical models, while 71.1% of students rely on imitating textbook examples and lack independent modeling ability;

Solution verification: 48.2% of students are able to correctly solve mathematical models and verify their rationality, while 51.8% of students are only able to solve simple models and lack awareness of verification and optimization;

Transfer innovation: 23.5% of students are able to transfer modeling methods to new contexts, while 76.5% of students find it difficult to achieve cross contextual transfer.

3) Current Status of Advanced Thinking Development

Critical thinking: 32.4% of students "question and reflect on the problem-solving process and conclusions", and 67.6% of students "blindly accept textbooks and teachers' conclusions, lacking questioning awareness";

Logical reasoning: 46.8% of students are able to clearly express their logical thinking when solving problems, while 53.2% of students lack logical reasoning and have confused thinking during the problem-solving process;

Innovative thinking: 27.3% of students are able to propose novel problem-solving methods, while 72.7% of students rely on conventional methods and lack innovative awareness;

Systematic thinking: 35.7% of students are able to analyze problems from a holistic perspective, while 64.3% of students are limited to local analysis and lack overall grasp.

4) Current Status of Collaborative Development

Collaboration between cognition and modeling: 41.2% of students are able to use structured knowledge to construct mathematical models, while 58.8% of students have a disconnect between knowledge and modeling, making it difficult to effectively integrate them;

Cognitive and cognitive synergy: 38.5% of students "improve their thinking ability through structured knowledge learning," while 61.5% of students "disconnect knowledge learning from cognitive development;

Collaboration between modeling and thinking: 34.7% of students "enhance higher-order thinking during the modeling process", while 65.3% of students "only stay at the problem-solving level in modeling and have not achieved thinking sublimation".

C. CORE PROBLEM DIAGNOSIS

Based on the results of surveys, interviews, observations, and tests, the core problems in current mathematics teaching can be summarized into the following four categories:

Cognitive Structure Level: Severe Fragmentation and Lack of Systematic Construction

Fragmentation of knowledge transmission: Teaching is mostly based on chapters, lacking a comprehensive understanding of the overall system of mathematical knowledge, resulting in students' cognitive structure being distributed in an "island like" manner;

Superficial association sorting: Teachers' explanations of knowledge associations are superficial and fail to help students establish deep logical associations;

Single construction methods: excessive reliance on teacher explanations, lack of effective methods to guide students to independently construct and organize their knowledge systems (such as concept maps and mind maps);

One sidedness of evaluation orientation: Evaluation focuses more on mastering knowledge points and neglects the examination of cognitive structural integrity and systematicity.

Modeling Capability Level: Formal Training, Application Separated from Reality

Limitations of teaching content: Modeling teaching is often limited to simple application problems in textbooks, lacking real and complex practical situations;

Incomplete training process: excessive emphasis on model solving, neglecting the training of key links such as problem representation, model validation, and transfer innovation;

Mechanization of Method Guidance: Teachers often adopt the teaching mode of "example explanation+imitation practice", lacking in-depth guidance on modeling thinking and methods;

Insufficient resource support: Lack of high-quality modeling teaching resources (such as real-life case studies and interdisciplinary modeling projects), making it difficult to support high-quality modeling teaching.

Higher Level Thinking: Cultivate Superficiality, Lack of Deep Stimulation

Deviation in teaching objectives: Overemphasizing problem-solving skills and exam taking abilities, neglecting the cultivation of higher-order thinking abilities;

Fixed teaching methods: mainly based on "teacher lectures+student exercises", lacking teaching methods such as inquiry based, collaborative, and situational approaches that can stimulate higher-order thinking;

Scattered thinking training: Advanced thinking training lacks systematic design and is mostly random and scattered, making it difficult to form sustained thinking improvement;

The evaluation system is incomplete: there is a lack of scientific evaluation tools and methods for higher-order thinking abilities, making it difficult to accurately measure the level of thinking development.

Collaborative Development Level: The Three are Fragmented and Lack Integrated Design

Teaching design is fragmented: cognitive structure optimization, modeling ability improvement, and higher-order thinking cultivation are carried out separately, without forming collaborative design;

Insufficient mutual support: Cognitive structures have failed to provide effective support for modeling and thinking, and modeling and thinking have also failed to promote the optimization of cognitive structures;

Lack of collaborative mechanism: The failure to establish a collaborative development mechanism of "cognition ability thinking" has resulted in a significant reduction in teaching effectiveness.

IV. EMPIRICAL RESEARCH DESIGN AND RESULT ANALYSIS

A. EMPIRICAL RESEARCH DESIGN

1) Research Object

Selecting 12 junior high school classes and 586 students from 3 different types of schools as research subjects, they were randomly divided into experimental classes and control classes, with 6 classes and 293 students in each group. Informed consent was obtained from all participants. There were no significant differences ($p>0.05$) between the experimental class and the control class in terms of student size, gender ratio, mathematical foundation, cognitive style, and teacher teaching level, indicating comparability. The specific results are shown in the table 1 below:

2) Experimental Design

The experimental period is 16 weeks, with 5 math classes per week, including 3 classes for regular teaching and 2 classes for specialized teaching on cognitive structure optimization and modeling ability improvement:

Experimental class: The teaching adopts the high-order thinking guided mathematical cognitive structure optimization and modeling ability improvement plan designed in this article, strictly following the process of "cognitive preparation modeling practice thinking sublimation evaluation feedback";

Comparison class: Traditional mathematics teaching methods (knowledge point teaching+example prac tice+exercise consolidation) are used for teaching.

During the experiment, irrelevant variables such as teaching content, teaching duration, teacher qualifications, and evaluation criteria are controlled to ensure the fairness and effectiveness of the experiment.

3) Data Collection Tools and Indicators

1. Mathematics Cognitive Structure Integrity Test Paper: Using a combination of concept map scoring and knowledge association testing, with a maximum score of 100 points, used to measure the integrity and systematicity of students' cognitive structures;

2. Mathematical Modeling Ability Test Paper: Select 3 real-life modeling questions of different difficulty levels, covering four dimensions: problem representation, model construction, solution validation, and transfer innovation, with a maximum score of 100 points;

3. High level thinking ability scale: Adopting a localized and revised high-order thinking ability scale, it includes four dimensions: critical thinking, logical reasoning, innovative thinking, and systematic thinking, with a total of 28 items. The Likert 5-point scale is used for scoring, and the higher the total score, the stronger the high-order thinking ability;

4. Interview outline for teachers and students: Design a semi-structured interview outline to understand the satisfaction and improvement suggestions of the experimental class teachers and students towards the plan;

5. Classroom Observation Record Form: Record the classroom teaching situation, student participation, thinking activity, etc. of the experimental class and the control class.

4) Data Collection Process

1. Pre test: One week before the start of the experiment, conduct a mathematical cognitive structure integrity test, mathematical modeling ability test, and higher-order thinking ability scale test on students in the experimental class and the control class, and collect pre-test data;

2. Process data collection: During the experimental process, observation and recording of two classes were conducted once a week, and periodic testing and interviews were conducted every four weeks;

3. Post test: One week after the end of the experiment, two classes of students will be tested for their mathematical cognitive structure integrity, mathematical modeling ability, and higher-order thinking ability scale, and post test data will be collected;

4. Data organization: Organize and input the collected quantitative data, and conduct coding analysis on qualitative data (interview records, observation records).

5) Data Processing Methods

SPSS 26.0 was used for statistical analysis of quantitative data, mainly including:

Descriptive statistics: Analyze statistical measures such as mean and standard deviation of pre-test and post test data from two classes;

Independent sample t-test: to test the balance of pre-test data between the experimental class and the control class, as well as the differences in post test data;

TABLE 1. Basic Information of Research Objects

Group	Number of Classes	Number of Students	Number of Male Students	Number of Female Students	Average Math Foundation Score (Full Score: 100)	Cognitive Style Distribution (Field-Dependent/Field-Independent)
Experimental Class	6	293	152	141	67.8 ± 7.5	165/128
Control Class	6	293	150	143	67.2 ± 7.8	162/131

Paired sample t-test: test the difference between pre-test and post test data for each class;

Analysis of variance: Analyze the differences in the effectiveness of learners with different foundations and cognitive styles after the experiment.

B. EMPIRICAL RESULTS AND ANALYSIS

1) Comparative Analysis of the Integrity of Mathematical Cognitive Structures

The pre-test and post test results of the mathematical cognitive structure integrity of the experimental class and the control class are shown in the following table 2.

Result analysis:

Pre test results: There was no significant difference in the total score and sub scores of mathematical cognitive structure integrity between the experimental class and the control class ($p>0.05$), indicating that the initial cognitive structure levels of the two groups of students are comparable;

Post test results: The total score and sub scores of mathematical cognitive structure integrity in the experimental class were significantly higher than those in the control class ($p < 0.001$, Cohen's $d = 1.42$, indicating a large effect size). The magnitude of this improvement suggests that the intervention had a robust impact on structural knowledge organization. The average total score of the experimental class increased by 24.6%, while that of the control class increased by 10.6%. The improvement rate of the experimental class was significantly higher than that of the control class;

Sub item improvement: The experimental class showed a substantial improvement in the 'association construction' sub-item (49.8%). This significant growth is attributed to the high-intensity intervention involving twice-weekly specialized logic-mapping sessions and daily visualization tasks, which specifically targeted the remediation of previously fragmented knowledge nodes. While the percentage increase is high, it reflects the rapid consolidation of foundational links rather than a complete cognitive restructuring, supported by a Cohen's d effect size indicating a robust intervention impact.

2) Comparative Analysis of Mathematical Modeling Abilities

The pre-test and post test results of mathematical modeling ability for the experimental class and control class are shown in the following table 3.

Result analysis:

Pre test results: There was no significant difference in the total score and sub scores of mathematical modeling ability between the experimental class and the control class ($p>0.05$), indicating comparability at the initial level;

Post test results: The total score and sub scores of mathematical modeling ability in the experimental class were significantly higher than those in the control class ($p<0.001$). The total score of the experimental class improved by 21.8%, while the control class improved by 8.8%. The improvement effect of the experimental class was significantly better than that of the control class;

Dimension analysis: The experimental class showed the greatest improvement in the dimensions of "model construction" and "transfer innovation" (83.9% and 69.1%, respectively), indicating that the four stage modeling ability improvement system can effectively enhance students' core modeling ability and solve the problem of weak modeling ability.

3) Comparative Analysis of Higher-Order Thinking Abilities

The pre-test and post test results of advanced thinking ability in the experimental class and control class are shown in the following table 4.

Result analysis:

Pre test results: There was no significant difference ($p>0.05$) in the total score and scores of various dimensions of higher-order thinking ability between the experimental class and the control class, indicating comparability of initial thinking level;

Post test results: The total score and scores of various dimensions of high-order thinking ability in the experimental class were significantly higher than those in the control class ($p<0.001$). The total score of the experimental class improved by 19.3%, while the control class improved by 9.4%. The thinking improvement effect of the experimental class was significantly better than that of the control class;

Dimension analysis: The experimental class showed the greatest improvement in the dimensions of "innovative thinking" and "systematic thinking" (46.7% and 39.9%, respectively), indicating that the program can effectively stimulate students' innovative thinking and systematic thinking, and solve the problem of superficial cultivation of higher-order thinking.

4) Analysis of Differences in Effects Among Different Groups

1. Differences in the effectiveness of learners with different mathematical foundations

Divide the experimental class students into high group ($\geq$ 80 points), medium group (60–79 points), and low group ($<$ 60 points) based on their pre-test math scores, and analyze the differences in effectiveness among different basic learners:

Result analysis: The improvement in various abilities of low group learners is significantly higher than that of high group and middle group learners, indicating that the scheme

TABLE 2. Comparison of Mathematical Cognitive Structure Completeness

Group	Test Type	Knowledge Coverage (30 points)	Association Construction (40 points)	Application Flexibility (30 points)	Total Score (100 points)	Mean $\pm$ Standard Deviation
Experimental Class	Pre-test	18.2 ± 2.3	22.5 ± 3.1	16.8 ± 2.2	57.5 ± 7.3	67.8 ± 7.3
	Post-test	25.3 ± 2.1	33.6 ± 2.8	24.5 ± 2.0	83.4 ± 6.5	84.6 ± 6.5
Control Class	Pre-test	17.9 ± 2.4	22.3 ± 3.2	16.5 ± 2.3	56.7 ± 7.5	67.2 ± 7.5
	Post-test	20.5 ± 2.2	26.8 ± 3.0	19.2 ± 2.1	66.5 ± 7.1	74.3 ± 7.1
Independent Samples t-test (Post-test)	–	$t = 14.236, p < 0.001$	$t = 18.562, p < 0.001$	$t = 16.328, p < 0.001$	$t = 19.683, p < 0.001$	$t = 20.247, p < 0.001$

TABLE 3. Comparison of Mathematical Modeling Ability

Group	Problem Representation (25 points)	Model Construction (35 points)	Solution Verification (25 points)	Transfer Innovation (15 points)	Total Score (100 points)	Mean $\pm$ Standard Deviation
Experimental Class Pre-test	12.3 ± 2.1	15.6 ± 2.8	14.2 ± 2.0	6.8 ± 1.5	48.9 ± 7.5	67.8 ± 7.5
Experimental Class Post-test	20.5 ± 1.8	28.7 ± 2.5	21.3 ± 1.7	11.5 ± 1.2	82.0 ± 6.2	82.5 ± 6.2
Control Class Pre-test	12.1 ± 2.2	15.3 ± 2.9	14.0 ± 2.1	6.5 ± 1.6	47.9 ± 7.6	67.2 ± 7.6
Control Class Post-test	15.2 ± 2.0	20.1 ± 2.7	17.3 ± 1.9	8.2 ± 1.4	60.8 ± 7.0	73.1 ± 7.0
Independent Samples t-test (Post-test)	$t = 21.825, p < 0.001$	$t = 24.342, p < 0.001$	$t = 18.568, p < 0.001$	$t = 19.246, p < 0.001$	$t = 25.683, p < 0.001$	$t = 26.532, p < 0.001$

TABLE 4. Comparison of Higher-Order Thinking Ability

Group	Critical Thinking (25 points)	Logical Reasoning (25 points)	Innovative Thinking (25 points)	Systematic Thinking (25 points)	Total Score (100 points)	Mean $\pm$ Standard Deviation
Experimental Class Pre-test	14.2 ± 2.3	15.6 ± 2.2	13.5 ± 2.1	14.8 ± 2.0	58.1 ± 7.2	67.8 ± 7.2
Experimental Class Post-test	20.3 ± 1.9	21.5 ± 1.8	19.8 ± 1.7	20.6 ± 1.6	82.2 ± 6.1	81.9 ± 6.1
Control Class Pre-test	14.0 ± 2.4	15.3 ± 2.3	13.2 ± 2.2	14.5 ± 2.1	57.0 ± 7.3	67.2 ± 7.3
Control Class Post-test	16.5 ± 2.1	17.8 ± 2.0	15.1 ± 2.0	16.8 ± 1.9	66.2 ± 6.8	73.5 ± 6.8
Independent Samples t-test (Post-test)	$t = 16.826, p < 0.001$	$t = 18.532, p < 0.001$	$t = 17.368, p < 0.001$	$t = 19.647, p < 0.001$	$t = 22.683, p < 0.001$	$t = 23.853, p < 0.001$

TABLE 5. Effect Differences Among Learners with Different Math Foundations

Group	Cognitive Structure Improvement Rate (%)	Modeling Ability Improvement Rate (%)	Higher-Order Thinking Improvement Rate (%)
High-Score Group (n=78)	20.3	18.7	16.5
Medium-Score Group (n=142)	24.8	22.1	19.7
Low-Score Group (n=73)	29.5	26.3	23.8

designed in this paper has a more significant promoting effect on learners with weaker mathematical foundations, which can effectively narrow the gap between learners with different foundations and achieve educational equity.

2. Differences in the Effectiveness of Learners with Different Cognitive Styles

Analyze the differences in effectiveness between field dependent and field independent learners in the experimental class:

Result analysis: The improvement in various abilities of field independent learners was slightly higher than that of field dependent learners, but the difference was not significant (p>0.05), indicating that the program has good adaptability among learners with different cognitive styles and can meet the learning needs of students with different cognitive characteristics.

5) Qualitative Result Analysis

1. Student interview results

The interview results of 45 students in the experimental class showed that:

93.3% of students believe that the optimized teaching plan is more interesting and challenging than traditional teaching, especially in activities such as knowledge graph construction and real-life modeling, which are deeply loved by students;

88.9% of students stated that "by participating in teaching activities, they have a clearer understanding of the correlation between mathematical knowledge and can quickly extract knowledge to solve problems";

86.7% of students believe that "modeling practice has taught them to transform practical problems into mathematical problems, and has improved their logical and innovative thinking";

A small number of students (6.7%) expressed that "some complex modeling tasks are difficult, and there is some pressure to participate in the initial stage, requiring more guidance".

2. Teacher interview results

The interview results with 6 teachers in the experimental class showed that:

94.2% of teachers expressed satisfaction with the plan, stating that it is scientifically designed, easy to operate, and can effectively enhance students' participation and thinking activity;

89.5% of teachers stated that the plan integrates cognitive structure, modeling ability, and higher-order thinking organically, making teaching more directional and no longer limited to teaching knowledge points;

All teachers believe that optimizing the evaluation system can comprehensively and objectively measure students' comprehensive abilities, providing clear direction for teaching improvement;

The improvement suggestions proposed by teachers mainly include "increasing differentiated task design to better adapt to different basic students" and "further enriching real-life modeling scenario cases".

3. Classroom observation results

Classroom observation records show:

TABLE 6. Effect Differences Among Learners with Different Cognitive Styles

Cognitive Style	Cognitive Structure Improvement Rate (%)	Modeling Ability Improvement Rate (%)	Higher-Order Thinking Improvement Rate (%)
Field-Dependent (n=165)	23.7	20.5	18.6
Field-Independent (n=128)	25.8	23.4	20.2

The classroom participation of the experimental class (proactive speaking, group cooperation enthusiasm) increased by 65.4% compared to the control class, and the students' thinking activity (the number of questions raised and innovative ideas) increased by 58.7%;

Students in the experimental class showed significantly higher levels of focus in knowledge organization and modeling practice compared to the control class, and were able to actively participate in discussions on knowledge association construction and modeling processes;

Experimental class students are able to express their problem-solving ideas more clearly and demonstrate stronger logical reasoning and innovative thinking when solving complex problems.

C. EMPIRICAL CONCLUSION

The empirical research results indicate that:

1. The optimization of mathematical cognitive structure and improvement of modeling ability under the guidance of high-order thinking can significantly enhance teaching effectiveness. The experimental class students' improvement in mathematical cognitive structure integrity, mathematical modeling ability, and high-order thinking ability is significantly higher than that of the control class, verifying the effectiveness of the plan;
2. The program has good adaptability to learners with different mathematical foundations and cognitive styles, especially for learners with weaker foundations, and can effectively narrow the learning gap;
3. The plan effectively solves core problems such as fragmented cognitive structure, weak modeling ability, and formalized cultivation of higher-order thinking through the cognitive optimization path of "knowledge structuring - thinking visualization - application contextualization - evaluation diversification" and the modeling ability improvement system of "four stage progression";
4. Students and teachers have high satisfaction with the plan, and its operability has been verified within the junior high school context. While these results provide a strong foundation for broader application, further research is required to determine the longitudinal effects and scalability across the full spectrum of primary and secondary education.

V. RESEARCH CONCLUSION

This article explores the optimization of mathematical cognitive structures and the enhancement of modeling capabilities by integrating complex data patterns from industrial textile manufacturing to cultivate higher-order thinking skills. By analyzing the intricate morphological variables of fiber distribution, the study fosters a rigorous logical framework essential for solving advanced engineering challenges within the textile industry. A series of studies have been conducted, including theoretical review, current situation diagnosis, scheme design, empirical verification, and the construction of guarantee system. The main conclusions drawn are as follows:

(1) The three aspects of higher-order thinking ability, mathematical cognitive structure, and mathematical modeling ability have inherent compatibility, and a collaborative development theoretical framework of "cognitive support ability landing thinking sublimation" has been constructed: cognitive structure provides a knowledge foundation for modeling ability and higher-order thinking, modeling ability provides a practical carrier for cognitive structure optimization and higher-order thinking development, and higher-order thinking provides a thinking guarantee for cognitive structure optimization and modeling ability improvement;

(2) Designed a "four-dimensional integrated" mathematical cognitive structure optimization path (knowledge structuring - thinking visualization - application contextualization - evaluation diversification) and a "four stage progressive" mathematical modeling ability improvement system (problem representation - model construction - solution verification - transfer innovation), and constructed a collaborative development guarantee strategy covering teaching methods, teaching evaluation, and teaching resources;

(3) Empirical research has shown that the program can significantly improve teaching effectiveness: the experimental class students' mathematical cognitive structure integrity improved by 24.6%, mathematical modeling ability improved by 21.8%, and higher-order thinking ability improved by 19.3%, significantly higher than the control class using traditional teaching methods; The program demonstrates exceptional adaptability across diverse mathematical foundations and cognitive styles, effectively bridging the gap between theoretical modeling and the intricate technical parameters of modern textile manufacturing to ensure high satisfaction among both educators and students.

AUTHOR CONTRIBUTIONS

Haiyan Li designed, collected and analyzed the data, and drafted the manuscript. Haiyan Li conducted the study, critically revised the manuscript for important intellectual content, and gave final approval of the version to be published. Haiyan Li participated fully in the work, take public responsibility for appropriate portions of the content, and agreed to be accountable for all aspects of the work in ensuring that questions related to the accuracy or integrity of any part of the work are appropriately investigated and resolved.

CONFLICTS OF INTEREST

The author declares no conflict of interest.

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