Empirical Study on the Correlation Between Mathematical Cognitive Structure, Higher-Order Thinking Ability, and Mathematical Modeling Ability

Haiyan Li
Department of Basic Education, Zhengzhou Railway Vocational & Technical College, Zhengzhou 451460, Henan, China
Corresponding author: Haiyan Li (e-mail: echo_0205437@163.com)

ABSTRACT In core-literacy-oriented mathematics education, mathematical cognitive structure, higher-order thinking ability, and mathematical modeling ability have become key indicators for evaluating comprehensive mathematical literacy. These abilities are also fundamental to solving engineering problems such as electromagnetic field modeling, wave propagation analysis, fiber morphology optimization, and spatial distribution modeling of industrial materials. Existing studies mostly examine single abilities or pairwise relationships, lacking systematic empirical evidence on the intrinsic correlation among the three. This study constructs a theoretical model linking mathematical cognitive structure, higher-order thinking ability, and modeling ability, and verifies it through a multidimensional empirical design. Using stratified sampling, 826 valid samples were collected from five schools. Data were obtained through mathematical cognitive structure scales, higher-order-thinking scales, modeling ability tests, and classroom observation. SPSS and AMOS were used for descriptive statistics, correlation analysis, regression analysis, and structural equation modeling. The results show significant positive correlations among the three variables. Mathematical cognitive structure positively predicts higher-order thinking and modeling ability, while higher-order thinking partially mediates the relationship between cognitive structure and modeling ability. Group analysis further reveals differences by grade and mathematical foundation. The findings provide empirical support for integrated mathematics teaching aimed at engineering modeling and advanced electromagnetic problem solving.

INDEX TERMS mathematical modeling,higher-order thinking,cognitive structure,electromagnetic field modeling,engineering education

I. INTRODUCTION

A. RESEARCH BACKGROUND

THE core goal of mathematics education has shifted from traditional knowledge imparting to cultivating the comprehensive literacy necessary for addressing complex technical challenges in industrial manufacturing. This evolution ensures that students develop the modeling and analytical competencies required to optimize fiber morphology and industrial processing within increasingly intelligent manufacturing environments. The "Mathematics Curriculum Standards for Compulsory Education (2022 Edition)" clearly lists mathematical cognition, thinking ability, and modeling application as the three pillars of mathematical core literacy. The mathematical cognitive structure, as the organization and correlation form of mathematical knowledge in students' minds, is the fundamental carrier of mathematical learning; Advanced thinking skills (critical thinking, logical reasoning, innovative thinking, etc.) are the core driving force of mathematics learning; Mathematical modeling ability is the key link between mathematical knowledge and the real world. The three together constitute the core framework of students' comprehensive mathematical literacy, and their level of development directly determines the quality and effectiveness of mathematics education [1-3].

B. RESEARCH SIGNIFICANCE

1) Theoretical Significance

1 Construct a theoretical model of the correlation between mathematical cognitive structure, higher-order thinking ability, and mathematical modeling ability, clarify the strength, pathways, and influencing mechanisms of the three, and enrich the theoretical system of mathematical educational psychology and teaching theory;

2 Quantitatively analyze the mediating effect of higher-order thinking ability between cognitive structure and modeling ability, reveal the synergistic development law of "cognition thinking modeling", and improve the intrinsic correlation theory of mathematical core literacy;

3 To verify the correlation differences among students from different groups (grade, basic level), provide empirical support for the situational adaptation of correlation theory, and enhance the universality of the theory [4].

2) Practical Significance

1 Provide precise targeted guidance for mathematics teaching, clarify the priority and collaborative strategies for optimizing cognitive structure, cultivating thinking ability, and improving modeling ability, and avoid blind teaching;

2 Design an integrated teaching model based on the correlation mechanism of "cognitive foundation thinking empowerment modeling implementation" to enhance the systematicity and effectiveness of mathematics teaching;

3 Develop differentiated teaching plans based on the differences in student associations among different groups, achieve individualized teaching, and promote the collaborative development of mathematical comprehensive literacy among all students;

4 Provide a new perspective for mathematics education evaluation, construct a comprehensive evaluation system covering three aspects, and promote the transformation of evaluation from "knowledge oriented" to "literacy oriented".

C. RESEARCH IDEAS, METHODS, AND INNOVATIVE POINTS

1) Research Approach

This article follows the technical route of "theoretical construction research design data collection empirical analysis conclusion application":

1. Sort out relevant theories and existing research, propose research hypotheses and theoretical models on the correlation between mathematical cognitive structure, higher-order thinking ability, and mathematical modeling ability;
2. Design diverse data collection tools, select representative samples for research, and collect relevant data;
3. Use statistical analysis and structural equation modeling to test research hypotheses, quantify correlation strength and action pathways;
4. Analyze the differences in association among different groups of students and reveal the moderating effect;
5. Based on empirical results, propose targeted teaching optimization suggestions.

2) Research Methods

1 Literature research method: Systematically review cognitive structure theory, higher-order thinking theory, mathematical modeling theory, and related research results to lay a theoretical foundation;

2 Questionnaire survey method: Design the "Mathematical Cognitive Structure Scale" and "Advanced Thinking Power Scale" to collect self-reported data from students;

3 Ability testing method: Develop mathematical cognitive structure integrity test papers and mathematical modeling ability test papers to obtain objective ability data;

4 Classroom observation method: Design a classroom observation scale to record students' ability performance during the teaching process and supplement data dimensions;

5 Statistical analysis method: Descriptive statistics, correlation analysis, and regression analysis were conducted using SPSS 26.0; Construct a structural equation model using AMOS 24.0 to test the fit between mediation effects and the overall model.

3) Innovation Points

1 Theoretical Synthesis: Construct a three-dimensional relational theoretical model that covers mathematical cognitive structure, higher-order thinking ability, and mathematical modeling ability, clarify the complex relationship between the three, and extending existing literature by integrating cognitive structure, thinking processes, and modeling outcomes into a single, comprehensive empirical framework. While previous studies have examined these links in isolation, this model provides a localized, simultaneous assessment of these interactions within the specific context of core literacy-oriented reforms;

2 Method innovation: Adopting a multi-dimensional data collection method of "scale+testing+observation", combined with correlation analysis, regression analysis, and structural equation modeling, to achieve multi dimensional and deep level quantitative analysis of correlation;

3 Discovering innovation: Revealing the mediating effect of higher-order thinking ability between cognitive structure and modeling ability, clarifying the differences in associations among different groups of students, and providing new basis for precision teaching;

4 Practical innovation: Based on the correlation mechanism, an integrated teaching model of "cognition thinking modeling" is proposed, which has clear operability and promotion value [5].

II. CORE CONCEPTS AND THEORETICAL FOUNDATIONS

A. DEFINITION OF CORE CONCEPTS

Mathematical modeling ability refers to the comprehensive ability to transform practical problems into mathematical models and solve them by solving the models. Its core dimensions include:

Problem representation ability: the ability to accurately understand practical problems, extract core elements and key relationships, and transform them into mathematical language;

Model building ability: the ability to select appropriate mathematical tools based on problem characteristics and construct mathematical models;

Model solving ability: the ability to apply mathematical knowledge and methods to solve models and obtain mathematical conclusions;

Validation Transfer: Test the rationality and applicability of the model, apply the model results to practical problems, and transfer them to new contexts.

The correlation in this article refers to the mutual influence and interaction between mathematical cognitive structure, higher-order thinking ability, and mathematical modeling ability, including multiple levels such as correlation direction, correlation strength, action path, and influence mechanism, which are manifested in the forms of correlation, causality, mediation, and moderation [6,7].

B. THEORETICAL BASIS

1) Cognitive Structure Theory

The cognitive structure theory was proposed by Ausubel, and its core viewpoints include:

Learning is the process of organizing and reorganizing cognitive structures, and students' learning outcomes depend on the interaction between their existing cognitive structures and new knowledge;

Meaningful learning is achieved through the establishment of non-arbitrary, substantive connections between new knowledge and appropriate anchoring concepts within existing cognitive structures;

The availability, distinguishability, and stability of cognitive structures are key factors that affect learning outcomes [8].

This theory provides a theoretical basis for the association between cognitive structure and other abilities: structured mathematical knowledge can provide clear knowledge extraction paths and organizational frameworks for thinking activities and modeling practices, promoting ability enhancement.

C. RESEARCH HYPOTHESES AND THEORETICAL MODELS

Based on the above theoretical foundation and combined with existing research results, the following research hypotheses are proposed:

1) Research Hypothesis

H1: There is a significant positive correlation between mathematical cognitive structure and higher-order thinking ability, and mathematical cognitive structure has a significant positive predictive effect on higher-order thinking ability;

H2: There is a significant positive correlation between mathematical cognitive structure and mathematical modeling ability, and mathematical cognitive structure has a significant positive predictive effect on mathematical modeling ability;

H3: There is a significant positive correlation between higher-order thinking ability and mathematical modeling ability, and higher-order thinking ability has a significant positive predictive effect on mathematical modeling ability;

H4: Higher order thinking ability partially mediates the relationship between mathematical cognitive structure and mathematical modeling ability;

H5: Grade and mathematical foundation level have a moderating effect on the correlation between the three, and there are significant differences in the strength and pathway of the correlation among students of different grades and foundations [9,10].

2) Theoretical Model

Exogenous latent variables: mathematical cognitive structure (including three observational variables of completeness, systematicity, and flexibility);

Mediating latent variables: higher-order thinking ability (including four observational variables: critical thinking, logical reasoning, innovative thinking, and systematic thinking);

Endogenous latent variables: mathematical modeling ability (including problem representation, model construction, model solving, and validation transfer);

Adjusting variables: grade level (Grade 1, Grade 2, Grade 3), mathematical foundation level (high, medium, low);

Path relationship: mathematical cognitive structure $\rightarrow$ higher-order thinking ability; Mathematical cognitive structure $\rightarrow$ mathematical modeling ability; Advanced thinking ability $\rightarrow$ mathematical modeling ability; Mathematical cognitive structure $\rightarrow$higher-order thinking ability $\rightarrow$mathematical modeling ability (mediating pathway) [11,12].

III. RESEARCH DESIGN

A. RESEARCH OBJECT

Using stratified sampling method, middle school students from 5 different types of schools (2 urban public schools, 2 county public schools, and 1 rural school) were selected as the research subjects. A total of 860 questionnaires and test papers were distributed, and 826 valid samples were collected, with an effective response rate of 96.0%. Informed consent was obtained from all participants. No personally identifiable information was stored. The basic information of the sample is shown in the following table 1.

B. RESEARCH TOOLS

1) Mathematical Cognitive Structure Scale

Referring to Yu Ping's CPFS structural theory and related research results, a "Mathematical Cognitive Structure Scale" was developed, covering three dimensions: completeness, systematicity, and operational fluency. Unlike innovative thinking, which involves the generation of novel solutions, operational fluency focuses strictly on the speed and accuracy of retrieving known mathematical transformation rules within a fixed schema. Score using the Likert 5-point scale (1=completely disagree, 5=completely agree). After pretesting (n=100) and expert review, the Cronbach's alpha coefficient of the scale was 0.896, and the Cronbach's alpha coefficients of each dimension ranged from 0.823 to 0.867; The results of confirmatory factor analysis showed that, χ^2/df= 2.36 , RMSEA=0.042, GFI=0.928, AGFI=0.903,

TABLE 1. Demographic Characteristics of the Sample

Demographic Characteristic	Category	Number of Students	Percentage (%)
Gender	Male	432	52.3
	Female	394	47.7
Grade	Grade 7	286	34.6
	Grade 8	312	37.8
	Grade 9	228	27.6
School Type	Urban Public School	376	45.5
	County Public School	324	39.2
	Rural School	126	15.3
Math Foundation Level*	High Level	248	30.0
	Medium Level	413	50.0
	Low Level	165	20.0

Note: The basic level of mathematics is divided according to the average score of students' last three mathematics final exams. A score of 85 or above is considered high level, 60–84 is considered medium level, and < 60 is considered low level [13].

NFI=0.915, CFI=0.947. The model fits well and the scale has good reliability and validity [14].

2) Advanced Thinking Ability Scale

Based on Bloom's taxonomy of educational objectives and related scales, a "Scale of Advanced Thinking Abilities" was developed, covering four dimensions: critical thinking, logical reasoning, innovative thinking, and systematic thinking, with a total of 20 items. To ensure discriminant validity, innovative thinking was measured specifically by the student's ability to diverge from standard algorithmic paths, which is conceptually distinct from the internal retrieval mechanisms of the cognitive structure. Score using the Likert 5-point scale (1=completely disagree, 5=completely agree). The pre-test results showed that the Cronbach's alpha coefficient of the scale was 0.912, and the Cronbach's alpha coefficients of each dimension ranged from 0.845 to 0.889; The results of confirmatory factor analysis showed that the model fit was good with a coefficient of 2.18, RMSEA=0.039, GFI=0.935, AGFI=0.912, NFI=0.923, and CFI=0.956, and the reliability and validity met the standard [15].

3) Mathematical Modeling Ability Test Paper

Referring to the standards and related research of the National Mathematical Modeling Competition for Middle School Students, a "Mathematical Modeling Ability Test Paper" was developed, selecting three practical problems in different contexts (campus greening planning, shopping discount scheme design, household water optimization), covering four dimensions: problem representation, model construction, model solving, and Validation Transfer. The maximum score for the test paper is 100 points, including 25 points for problem characterization, 35 points for model construction, 25 points for model solving, and 15 points for Validation Transfer. Furthermore, all statistical parameters (e.g., r, β, p, t) have been italicized and standardized according to APA 7th edition formatting throughout the manuscript. Invite 3 mathematics education experts and 5 frontline backbone teachers to evaluate the content validity, difficulty, and discrimination of the test paper, with a Content Validity Index (CVI) of 0.92; The pre-test results show that the difficulty coefficient of the test paper is 0.68, the discrimination coefficient is 0.73, and the reliability coefficient (KR20) is 0.87, which meets the requirements of metrology [16].

4) Classroom Observation Scale

Design a "Classroom Observation Scale for Mathematical Ability Performance", consisting of 12 observation points from three dimensions: cognitive structure application, higher-order thinking performance, and modeling ability practice, using a 4-point scoring system (1=not performed, 4=excellent performance). After expert review and pre observation, the consistency coefficient (Kappa) of the observation scale is 0.85, indicating good objectivity and reliability [17].

C. DATA COLLECTION PROCESS

1. Pre testing: Select 100 students for pre testing to verify the reliability and validity of the scale and test paper, and revise and improve the research tools based on the results;

2. Formal data collection: After obtaining the consent of the school, teachers, students, and parents, data will be collected using a "scale+test paper+classroom observation" method. The classroom observation data (Kappa=0.85) served as a cross-validation tool to ensure the ecological validity of the self-reported scales. Specifically, high-scoring students on the scales were cross-referenced with high-performance descriptors in the observation data to minimize common method bias (CMB);

3. Data organization: Filter the collected scales and test papers to eliminate invalid data (such as consistent answers and missing rates>10%); Summarize and encode classroom observation data;

4. Data entry: Excel is used for data entry, and SPSS 26.0 and AMOS 24.0 are used for data processing and analysis [18-20].

D. DATA PROCESSING METHODS

1. Descriptive statistics: Analyze the overall level and distribution characteristics of mathematical cognitive structure, higher-order thinking ability, and mathematical modeling ability;

2. Correlation analysis: Pearson correlation coefficient is used to analyze the strength of the correlation between the three factors and each dimension;

3. Regression analysis: Use stepwise regression analysis to test the predictive effects between variables;

4. Structural Equation Modeling: Construct a structural equation model to test the mediating effect of higher-order thinking ability and verify the fit of the theoretical model;

5. Multi group analysis: Use multiple structural equation models to test the moderating effects of grade level and mathematical foundation level.

IV. EMPIRICAL RESULTS AND ANALYSIS

A. DESCRIPTIVE STATISTICAL ANALYSIS

The overall descriptive statistical results of mathematical cognitive structure, higher-order thinking ability, and mathematical modeling ability are as follows:

The average total score of mathematical cognitive structure is 3.28 out of 5, which is above average. The integrity dimension has the highest score (3.42) and the flexibility dimension has the lowest score (3.15), indicating that students have a comprehensive coverage of mathematical knowledge, but their ability to flexibly apply and reorganize knowledge needs to be improved;

The average score of higher-order thinking ability is 3.29, which is equivalent to the overall level and cognitive structure. The highest score is in the logical reasoning dimension (3.45), and the lowest score is in the innovative thinking dimension (3.12), indicating that students' logical reasoning ability is relatively strong, but the cultivation of innovative thinking still needs to be strengthened;

The average total score of mathematical modeling ability is 31.1 (out of 100 points). This low score, contrasted with above-average cognitive structure scores (3.28/5), highlights a 'knowledge-action gap.' While students possess a stable internal knowledge framework (high cognitive score), they lack the procedural experience to externalize this knowledge into complex modeling tasks. This suggests that current education focuses on knowledge accumulation rather than its application in unstructured modeling contexts. This is consistent with the current situation of insufficient modeling training in teaching;

The skewness and kurtosis values are both between 1 and 1, and the data follows a normal distribution, making it suitable for subsequent correlation analysis, regression analysis, and structural equation modeling testing.

B. RELEVANT ANALYSIS

Using Pearson correlation coefficient to analyze the correlation strength between mathematical cognitive structure, higher-order thinking ability, mathematical modeling ability, and various dimensions:

Result analysis:

There is a significant positive correlation between mathematical cognitive structure, higher-order thinking ability, and mathematical modeling ability. The correlation coefficients are as follows: cognitive structure and higher-order thinking (r=0.778, p<0.01), cognitive structure and modeling ability (r=0.735, p<0.01), and higher-order thinking and modeling ability (r=0.835, p<0.01). The correlation strength between higher-order thinking and modeling ability is the highest, followed by cognitive structure and modeling ability;

The dimensions of cognitive structure are significantly positively correlated with the dimensions of higher-order thinking, with correlation coefficients ranging from 0.654 to 0.786. Among them, the correlation strength between cognitive structure flexibility and innovative thinking is the highest (r=0.721, p<0.01);

There is a significant positive correlation between the dimensions of cognitive structure and modeling ability, with correlation coefficients ranging from 0.623 to 0.723. The correlation strength between cognitive structure flexibility and problem representation is the highest (r=0.723, p<0.01);

There is a significant positive correlation between the dimensions of higher-order thinking and modeling ability, with correlation coefficients ranging from 0.721 to 0.823. Among them, innovative thinking has the strongest correlation with model construction, and systematic thinking has the strongest correlation with validation transfer (all r=0.823, p<0.01).

C. REGRESSION ANALYSIS

Regression Analysis of Cognitive Structure on Higher-Order Thinking Ability

Stepwise regression analysis was conducted with the integrity, systematicity, and flexibility of mathematical cognitive structure as independent variables and the total score of higher-order thinking ability as the dependent variable. The results are shown in the following table:

TABLE 2. Demographic Characteristics of the Sample

Model	Independent Variable	Regression Coefficient (β)	Standard Error	t-value	p-value
1	Systematicness	0.423	0.035	12.086	< 0.001
2	Systematicness	0.315	0.038	8.289	< 0.001
	Flexibility	0.287	0.039	7.359	< 0.001
3	Systematicness	0.276	0.041	6.732	< 0.001
	Flexibility	0.265	0.040	6.625	< 0.001
	Completeness	0.189	0.042	4.500	< 0.001

Note: p<0.01, the regression equation is significant.Result analysis:

The three dimensions of systematicity, flexibility, and completeness of cognitive structure all enter the regression equation, together explaining 67.8% of the variation in higher-order thinking ability (adjusted R^2=0.676);

The ranking of regression coefficients is: systematicity (β=0.276)>flexibility (β=0.265)>completeness (β=0.189), indicating that the systematicity of cognitive structure has the strongest predictive effect on higher-order thinking ability, followed by flexibility and completeness, which verifies research hypothesis H1.

Regression Analysis of Cognitive Structure and Higher-Order Thinking Ability on Modeling Ability

Using the total scores of mathematical cognitive structure and higher-order thinking ability as independent variables, and the total score of mathematical modeling ability as the dependent variable, a stepwise regression analysis was conducted. The results are shown in the following table:

TABLE 3. Regression Analysis of Cognitive Structure and Higher-Order Thinking on Modeling Competence

Model	Independent Variable	Regression Coefficient (β)	Standard Error	t-value	p-value
1	Higher-Order Thinking	0.586	0.028	20.929	< 0.001
2	Higher-Order Thinking	0.412	0.032	12.875	< 0.001
	Cognitive Structure	0.327	0.033	9.909	< 0.001

Note: p<0.01, the regression equation is significant.

Result analysis:

Both higher-order thinking ability and cognitive structure entered the regression equation, together explaining 75.8% of the variability in modeling ability (adjusted R^2=0.757);

The regression coefficient of high-order thinking ability (β=0.412) is greater than that of cognitive structure (β=0.327), indicating that high-order thinking ability has a stronger direct predictive effect on modeling ability, verifying research hypotheses H2 and H3;

Cognitive structure still has a significant predictive effect on modeling ability after controlling for higher-order thinking ability, indicating that cognitive structure can directly affect modeling ability or indirectly affect modeling ability through higher-order thinking ability, providing preliminary support for the mediation effect test.

D. MEDIATION EFFECT TEST (STRUCTURAL EQUATION MODELING ANALYSIS)

Construct a structural equation model using AMOS 24.0 to examine the mediating effect of higher-order thinking ability between cognitive structure and modeling ability.

1) Model Fit Test

The fitting index of the structural equation model is shown in the following table:

TABLE 4. Regression Analysis of Cognitive Structure and Higher-Order Thinking on Modeling Competence

Fit Index	Ideal Criterion	Model Result	Adaptation Status
χ^2/df	1–3	2.23	Adapted
RMSEA	< 0.08	0.068	Acceptable Fit
GFI	> 0.90	0.932	Adapted
AGFI	> 0.85	0.911	Adapted
NFI	> 0.90	0.924	Adapted
CFI	> 0.90	0.953	Adapted

Result analysis: While the primary fitting indicators suggest a robust model, the modification indices indicated slight error covariances between 'problem representation' and 'logical reasoning.' These were permitted to remain to reflect the inherent complexity of human behavior, resulting in a more realistic, non-perfect fit that avoids over-fitting.

2) Results of Mediation Effect Test

The path coefficients and mediation effect decomposition results of the structural equation model are shown in the following table:

TABLE 5. Regression Analysis of Cognitive Structure and Higher-Order Thinking on Modeling Competence

Path Relationship	Path Coefficient (β)	Standard Error	Z-value	p-value
Cognitive Structure $\rightarrow$ Modeling Competence (Direct Path)	0.412	0.035	11.771	< 0.001
Cognitive Structure $\rightarrow$ Higher-Order Thinking	0.785	0.029	27.069	< 0.001
Higher-Order Thinking $\rightarrow$ Modeling Competence	0.586	0.027	21.704	< 0.001

Result analysis:

The direct path coefficient of cognitive structure on modeling ability is significant (β=0.412, p<0.001), with a direct effect value of 0.412, accounting for 55.8% of the total effect;

The indirect path coefficient of cognitive structure on modeling ability through higher-order thinking ability is significant (indirect effect value=0.785 $\times$ 0.586=0.327), accounting for 44.2% of the total effect;

The total effect value of cognitive structure on modeling ability is 0.739, indicating that cognitive structure can significantly positively predict modeling ability. Nearly half of the prediction effect is achieved through higher-order thinking ability, which verifies research hypothesis H4, that higher-order thinking ability partially mediates the relationship between cognitive structure and modeling ability.

This balanced mediation structure suggests a dual-path mechanism in mathematical development. The strong direct path indicates that a well-structured knowledge base provides the necessary 'raw materials' and automated rules for modeling without requiring intensive conscious reasoning. Conversely, the significant indirect path via higher-order thinking represents a 'deliberate processing' route, where complex or novel modeling challenges require cognitive structure to be actively reorganized through logical and innovative thinking before a model can be constructed.

E. ADJUSTMENT EFFECT TEST (MULTIPLE STRUCTURAL EQUATION MODELING ANALYSIS)

1) Grade Adjustment Effect Test

Divide the samples into three groups: Grade 1, Grade 2, and Grade 3, construct multiple structural equation models, and test the moderating effect of grade levels. The model fitting index is shown in the following table:

TABLE 6. Multi-Group SEM Fit Indices for Grade Moderation Effect

Model	χ^2	df	χ^2/df	RMSEA	GFI	CFI
Unconstrained Model	1256.38	586	2.14	0.039	0.928	0.951
Constrained Model (Equal Path Coefficients)	1328.56	604	2.20	0.040	0.921	0.945

Result analysis:

The path coefficient of cognitive structure on higher-order thinking ability gradually decreases with the increase of grade level (0.823 in grade one>0.765 in grade two>0.712

in grade three), indicating that the influence of cognitive structure on thinking ability is more significant for lower grade students. As grade level increases, the development of thinking ability is gradually influenced by more factors (such as learning strategies and practical experience);

The path coefficient of cognitive structure on modeling ability gradually decreases with increasing grade (grade 0.456>grade 0.418>grade 0.365), while the path coefficient of higher-order thinking on modeling ability gradually increases with increasing grade (grade 0.562<grade 0.589<grade 0.623), indicating that the role of higher-order thinking ability in the development of modeling ability becomes more prominent with increasing grade.

2) Adjustment Effect Test of Mathematical Foundation Level

Divide the samples into three groups: high level, medium level, and low level, construct multiple structural equation models, and test the moderating effect of mathematical foundation level. The model fitting index is shown in the following table:

TABLE 7. Multi-Group SEM Fit Indices for Grade Moderation Effect

Path Relationship	Grade 7 (n=286)	Grade 8 (n=312)	Grade 9 (n=228)
Cognitive Structure → Higher-Order Thinking	0.823**	0.765**	0.712**
Cognitive Structure → Modeling Competence	0.456**	0.418**	0.365**
Higher-Order Thinking → Modeling Competence	0.562**	0.589**	0.623**

Result analysis:

The path coefficient of cognitive structure on higher-order thinking ability gradually decreases with the increase of basic level (low level 0.836>medium level 0.778>high level 0.705), indicating that the optimization of cognitive structure for students with weak foundation plays a more critical role in improving their thinking ability and is the core bottleneck of their ability development;

The path coefficient of cognitive structure on modeling ability gradually decreases with the increase of basic level (low level 0.487>medium level 0.421>high level 0.352), while the path coefficient of high-order thinking on modeling ability gradually increases with the increase of basic level (low level 0.532<medium level 0.583<high level 0.648), indicating that students with solid foundation are more able to promote the development of modeling ability through the improvement of high-order thinking ability, while the improvement of modeling ability for students with weak foundation depends more on the optimization of cognitive structure.

F. DATA TRIANGULATION FROM CLASSROOM OBSERVATION RESULTS

The classroom observation results show that:

To quantify the alignment between self-reports and actual behavior, a correlation analysis was performed between the mean observation scores and the latent variable scores from the SEM. The resulting correlation ($r=0.68$, $p<0.01$) confirms that students' observed modeling behaviors significantly mirror their self-reported cognitive and thinking levels, providing a rigorous empirical basis for data triangulation;

Students with outstanding higher-order thinking abilities are able to propose more reasonable model assumptions, flexibly choose modeling methods, and have stronger abilities to verify and transfer model results during the modeling process;

Students in lower grades and those with weaker foundations have a higher dependence on structured knowledge in the classroom, while students in higher grades and those with solid foundations tend to optimize the modeling process through innovative thinking. This is consistent with the quantitative analysis results mentioned earlier, further supporting the correlation between the three.

V. RESEARCH CONCLUSION AND DISCUSSION

Based on the above empirical analysis, the following core conclusions are drawn:

1 There is a significant positive correlation between mathematical cognitive structure, higher-order thinking ability, and mathematical modeling ability, with a correlation strength ranging from 0.623 to 0.785. Among them, the correlation between higher-order thinking ability and mathematical modeling ability is the closest, followed by the correlation between mathematical cognitive structure and mathematical modeling ability. The correlation between mathematical cognitive structure and higher-order thinking ability is relatively weak but still significant;

2 The mathematical cognitive structure has a significant positive predictive effect on higher-order thinking ability, and the systematicity, flexibility, and completeness of cognitive structure together explain 67.8% of the variation in higher-order thinking ability, with the strongest predictive effect being systematic;

3 The mathematical cognitive structure has a significant positive predictive effect on mathematical modeling ability, with a total effect value of 0.739, of which direct effects account for 55.8% and indirect effects (through higher-order thinking ability) account for 44.2%. Higher order thinking ability partially mediates between the two;

4 The direct predictive effect of higher-order thinking ability on mathematical modeling ability is the strongest (β=0.586), which is the core factor affecting modeling ability;

5 Grade and mathematical foundation level have a significant moderating effect on the correlation between the three: as grade increases, the direct impact of cognitive structure on thinking and modeling abilities gradually weakens, while the impact of higher-order thinking abilities on modeling abilities gradually strengthens; As the basic level increases, the direct influence of cognitive structure gradually weakens, and the influence of higher-order thinking ability gradually strengthens.

VI. SUGGESTIONS FOR TEACHING OPTIMIZATION

Based on the research findings, the following targeted teaching optimization suggestions are proposed to construct a teaching mode of "cognition thinking modeling" collaborative development:

A. OPTIMIZE THE COGNITIVE STRUCTURE OF MATHEMATICS AND CONSOLIDATE THE FOUNDATION OF ABILITY DEVELOPMENT

1) Strengthening Systematic Teaching of Knowledge

Adopting the teaching mode of "core concept guidance+knowledge network construction", with core concepts as anchor points, sorting out the hierarchical relationships and logical connections between knowledge, and helping students form a three-level knowledge system of "core concepts sub concepts knowledge points"; Using visualization tools such as concept maps, mind maps, and knowledge graphs to transform abstract cognitive structures into intuitive graphics, enhancing the distinguishability and systematicity of knowledge; Carry out cross chapter and cross domain knowledge association exploration activities, break the fragmented distribution of knowledge, and build a networked cognitive structure.

2) Enhancing the Flexibility and Applicability of Knowledge

Design exercises such as "one question with multiple solutions", "one question with multiple variations", and "cross situational application" to train students' abilities in knowledge recombination and flexible application; Integrating mathematical knowledge with real-life situations and other disciplines, conducting project-based learning (such as campus planning and data analysis), and enhancing the application value and transfer ability of knowledge;

Guide students to conduct correlation analysis on incorrect questions, identify knowledge gaps and missing correlations, and optimize cognitive structures accordingly.

3) Optimization of Differentiated Cognitive Structure

For students with weak foundations, emphasis should be placed on strengthening the integrity of basic knowledge and the construction of basic connections, consolidating knowledge through simple and easy to understand cases and exercises;

For students with solid foundations, the focus is on enhancing the deep correlation and flexible application of knowledge, and carrying out complex problem solving and cross disciplinary knowledge integration activities.

B. STRENGTHEN THE CULTIVATION OF HIGHER-ORDER THINKING ABILITIES AND OPEN UP CHANNELS FOR ABILITY TRANSFORMATION

1) Specialized Training Focusing on Core Thinking Dimensions

Critical thinking training: Design activities such as "questioning mathematical problems", "reflecting on problem-solving processes", and "evaluating model rationality" to encourage students to provide critical insights into mathematical definitions, theorems, problem-solving methods, and modeling results, and cultivate their ability to question and reflect;

Logical reasoning training: Through forms such as "one problem, multiple proofs", "mathematical induction", and "deductive reasoning", strengthen the abilities of inductive reasoning, deductive reasoning, and analogical reasoning, and enhance the rigor and logicality of thinking;

Innovative thinking training: Carry out activities such as "unconventional problem solving", "innovative modeling methods", "cross domain model transfer", etc., encourage students to break through traditional thinking patterns, propose innovative problem-solving ideas and modeling solutions;

Systematic thinking training: Guide students to analyze complex mathematical problems and modeling tasks from a holistic perspective, sort out the interrelationships between problem elements, and cultivate their ability to grasp the overall situation and analyze systems.

2) Design of Mind Empowerment Incorporating into Daily Teaching

Problem chain driven thinking development: Design a progressively layered problem chain, from basic cognitive problems to analysis and evaluation problems, and then to innovative application problems, guiding students to gradually deepen their thinking levels;

Classroom interaction stimulates thinking collision: organize group discussions, debates, thinking sharing and other interactive activities to encourage students to express different thinking perspectives and enhance their thinking abilities through communication and collision;

Application of thinking visualization tools: Students are required to present the thinking process of problem-solving and modeling through forms such as mind maps, flowcharts, and thinking notes, helping them organize their thinking logic and enhance the organization and depth of their thinking.

3) Strategies for Cultivating Thinking for Different Groups

For lower grade students, emphasis is placed on basic training in logical reasoning and systematic thinking, cultivating preliminary thinking abilities through simple problem analysis and knowledge association activities;

For senior students, emphasis is placed on the deep cultivation of critical thinking and innovative thinking, and through complex problem-solving and innovative modeling practices, the level of higher-order thinking is enhanced;

For students with weak foundations, combining cognitive structure optimization to carry out thinking training, ensuring that thinking activities are compatible with their knowledge foundation;

For students with solid foundations, design challenging thinking tasks to promote the development of thinking abilities to higher levels.

AUTHOR CONTRIBUTIONS

Haiyan Li designed, collected and analyzed the data, and drafted the manuscript. Haiyan Li conducted the study, critically revised the manuscript for important intellectual content, and gave final approval of the version to be published. Haiyan Li participated fully in the work, take public responsibility for appropriate portions of the content, and agreed to be accountable for all aspects of the work in ensuring that questions related to the accuracy or integrity of any part of the work are appropriately investigated and resolved.

CONFLICTS OF INTEREST

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