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Design and Stability Analysis of Adaptive Synchronization Control Strategies for Integer and Fractional Chaotic Systems

Xiaoqing Zhang, Peng Zhang
Department of Science, Taiyuan Institute of Technology, Taiyuan 030008, Shanxi, China
Corresponding author: Xiaoqing Zhang (e-mail: zhangxqty@163.com)

ABSTRACT Chaotic synchronization plays an important role in nonlinear dynamics, secure communications, and complex information systems, yet parameter uncertainty remains a major obstacle to practical implementation. This study proposes a unified adaptive synchronization control strategy for both integer-order and fractional-order chaotic systems under unknown parameter conditions. Based on Lyapunov stability theory, generalized controller structures and adaptive parameter update laws are designed to achieve robust synchronization independent of specific system forms. Stability analysis theoretically proves asymptotic synchronization and bounded convergence of parameter estimation errors. Numerical simulations using representative Lorenz and fractional-order Chen systems verify the effectiveness and robustness of the proposed method. Results demonstrate rapid synchronization error convergence and accurate parameter estimation under uncertain conditions. The proposed framework provides theoretical support for nonlinear control systems and offers application potential in secure communications, electromagnetic signal transmission, and complex dynamic network synchronization.

INDEX TERMS chaotic synchronization, adaptive control, fractional-order systems, secure communications, electromagnetic signal transmission

I. INTRODUCTION

CHAOS as a complex nonlinear dynamic behavior is ubiquitous in nature as well in artificially created systems. Since late last century, chaos control and synchronization has slowly become one of the cutting-edge areas of research in the field of nonlinear science. Chaotic synchronization technology has received widespread attention due to its potential in establishing secure communication links and optimizing neural network performance, while also providing a sophisticated framework for synchronizing the high-speed nonlinear motions of multiple spindles and rollers in precision machinery. This synchronization capability ensures the consistent physical properties of advanced functional materials by accurately simulating and controlling the complex, coupled dynamics inherent in modern manufacturing processes. The heart of the issue of chaotic synchronization is the design of effective control strategies in which two or more uncorrelated and therefore independent chaotic systems can evolve together.

In practical applications, it is often difficult to obtain accurate mathematical models of chaotic systems, and the system parameters can have uncertainty due to variations of the environment, aging of the devices, or errors in the modeling. This requires the designed synchronization controller to be equipped with the ability to adjust to system changes in an online manner, i.e. adaptive capability. In the last several years, various adaptive synchronization control schemes of different types of chaotic systems were proposed by various researchers. Some special studies have been conducted on the dynamic analysis and controller design of certain chaotic systems in order to realize synchronization, through the construction of nonlinear functions or the introduction of sliding mode control technology. However, most of these methods have been designed for integer order chaotic systems, and their design process is highly coupled with the equations of the system under study, which makes it hard to extend directly to fractional order systems. Fractional calculus, because of its nonlocality and memory effect, is more able to describe the some real-world physical processes and material properties. Synchronization control using fractional-order chaotic systems has great significance in the complex signal control and the model study for novel materials. Therefore, it has a high theoretical and practical value to develop a unified adaptive synchronization control framework for integer-order and fractional-order chaotic systems.

To address limitations in existing research regarding system universality, this paper proposes an adaptive synchronization control strategy for integer-order and fractional-order chaotic systems to precisely regulate the nonlinear dynamics of high-speed material processing and material tensioning processes. By integrating these advanced control laws, the strategy ensures the stable synchronization of complex machinery components under varying operational conditions, thereby enhancing the consistency and quality of high-performance technical materials. This strategy solution, through the building of unified controller structure and parameter adaptive law, aims to solve synchronization problem of a class parameter uncertain chaotic systems and theoretically guarantee the system stability. The organization of the paper is as follows: Part II summarizes the research development in the related area; Part III details the control method that is proposed, comprising of controller design and stability proof; Part IV confirms the effectiveness of the proposed control method based on numerical simulation and elaborates on the simulation results around the control method; and Part V summarizes the entire work.

II. RELATED WORK

Synchronization control of chaotic systems is a research hotspot in the field of nonlinear science and has broad application prospects in engineering fields such as secure communication, neural networks, and biomedicine. Mohadeszadeh and Pariz[1] proposed an adaptive synchronization scheme to drive chaotic systems with uncertain parameters and realize synchronization between the transmitter and receiver. Zhang et al. [2] constructed and analyzed a novel multi-vortex chaotic system based on nonlinear functions, then designed an adaptive synchronization controller and implemented it in FPGA hardware. Huang[3] studied a high-performance control algorithm based on parameter adaptation and adjusted the controller parameters online to adapt to system changes. Dousseh et al. [4] proposed a modified chaotic financial system model and designed an adaptive controller to realize the synchronization and anti-synchronization control of the system. Kobiolka et al. [5] compared the performance of traditional reduced-order adaptive synchronization control method and data-driven method based on machine learning in realizing neuron synchronization. Ramar and Mohanavel[6] constructed a new chaotic system that can generate five-vortex and two-vortex systems and studied its offset enhancement and amplitude control characteristics. Amira and Hannachi[7] performed dynamic analysis on three-dimensional chaotic systems, including equilibrium points, bifurcation diagrams and Lyapunov exponents , and then designed an adaptive controller to achieve synchronization of the system. Huang et al. [8] combined adaptive control with sliding mode control to achieve fast and accurate synchronization of fractional-order memristor chaotic systems. Hidden multi-vortex memristor chaotic systems are more difficult to detect because they have no equilibrium points or only stable equilibrium points. Lai et al. [9] performed dynamic analysis on such systems and proposed a fixed-time synchronization control strategy. Singh et al. [10] explored the use of chaotic control theory to solve biomedical problems . The above studies have made progress in adaptive synchronization control of chaotic systems, but most methods are designed for specific chaotic systems and lack a control framework that is uniformly applicable to integer-order and fractional-order systems. To address the aforementioned limitations, this paper proposes an adaptive synchronization control strategy for integer-order and fractional-order chaotic systems. By constructing a unified adaptive law and controller structure, synchronization control of both types of systems is achieved. This research aims to provide a more universal and theoretically profound solution for chaotic system synchronization.

III. METHODS

A. PROBLEM DESCRIPTION AND SYSTEM MODEL CONSTRUCTION

This study considers a class of chaotic systems with uncertain parameters as driving systems, whose state equations can be described in a general form. This form can cover common integer-order chaotic systems and can also be naturally extended to the description of fractional-order chaotic systems by introducing fractional-order differential operators. Under this unified description framework, the dynamic behavior of the system is determined by its structure function and a set of unknown constant parameters. Correspondingly, a response system with the same structure but independent and equally unknown parameters is constructed, and its state variables will follow the trajectory of the driving system. The synchronization between the two systems depends on a control input attached to the response system. The control objective is to design a suitable controller so that the state of the response system asymptotically approaches the state of the driving system, regardless of the specific values of the system parameters. Under this framework, the initial states of the driving system and the response system are set to different values to simulate the initial mismatch state when two independent systems start up in practical applications. The synchronization error is defined as the difference between the state of the driving system and the state of the response system. Then the control objective is transformed into designing a control law to drive the synchronization error vector to converge to zero when time approaches infinity [11-12] .

B. ADAPTIVE CONTROLLER STRUCTURE DESIGN

To achieve the above control objectives, this paper proposes a structured adaptive controller design method. The core idea of this method is to use the structural information of the system to construct a control input containing adjustable gain and parameter estimates. The controller is mainly composed of two parts: one part is used to offset the nonlinear dynamic differences caused by unknown parameters in the drive system and the response system, which is called the compensation term; the other part is used to perform negative feedback adjustment on the synchronization error itself, which is called the stabilization term. The form of the compensation term is closely related to the structure function of the system. It aims to eliminate the root cause of the synchronization error divergence from the structure by introducing dynamic compensation related to the unknown parameter estimates. The stabilization term adopts the classic linear state feedback form, and its feedback gain matrix determines the speed of error convergence. The unknown parameters in the controller are not directly given, but are updated online in real time by a set of carefully designed adaptive laws [13] .

For integer-order systems, the parameter adaptive law is designed as follows:

$$\hat{\theta} = -G^T(x)e, \quad \hat{\phi} = G^T(y)e \tag{1}$$

$\hat{\theta}$ is the estimated value of the driving system's unknown parameters θ; $G^T(x)$ is the transpose of the coefficient matrix $G(x)$, used to feed the synchronization error back into the parameter update process; e is the synchronization error vector; $\hat{\phi}$ is the estimated value of the response system's unknown parameters ϕ; and $G^T(y)$ is the transpose of the coefficient matrix $G(y)$.

For fractional-order systems, the corresponding fractional-order parameter adaptive laws are designed as follows:

$$D_t^q\hat{\theta} = -G^T(x)e, \quad D_t^q\hat{\phi} = G^T(y)e \tag{2}$$

$D_t^q\hat{\theta}$ and $D_t^q\hat{\phi}$ refer, respectively, to the fractional-order derivatives of the estimated parameters of the driving system and the response system.

The design of these adaptive laws is the key to the method. It must ensure the stability of the entire closed-loop system and make the parameter estimates converge to near their true values.

C. PARAMETER ADAPTIVE LAW AND STABILITY PROOF FRAMEWORK

The design of the adaptive law follows Lyapunov's stability theory. First, based on the synchronization error dynamics and parameter estimation error, a scalar function, the Lyapunov function, is constructed, containing the energy of the system state error and the energy of the parameter estimation error. This function is positive definite, reflecting the degree to which the system deviates from the synchronization state and the parameters deviate from their true values. Then, the derivative of this function is calculated along the trajectory of the closed-loop system. By cleverly designing the form of the adaptive law, the derivative can be either negative definite or semi-negative definite. When the derivative is negative definite, it indicates that the Lyapunov function value strictly decreases with time, thus ensuring that both the synchronization error and the parameter estimation error asymptotically converge to zero. When the derivative is semi-negative definite, it indicates that the Lyapunov function value is non-increasing. Combining this with the system's dynamic characteristics, the Lassalle invariance principle can be used to prove that the synchronization error still asymptotically converges to zero, while the parameter estimates converge to a constant value related to the initial conditions. The adaptive law designed in this study is based on this principle; it uses the synchronization error signal to drive the update of the parameter estimates, ensuring that the direction of parameter estimation always points in the direction that reduces the synchronization error. The entire design process does not depend on the specific parameter values of the system, but only on its structural form, thus ensuring the universality of the method for a class of chaotic systems with uncertain parameters [14-15]. Through the above steps, an adaptive synchronous control system integrating a controller and a parameter estimator is constructed, and its stability is theoretically guaranteed.

IV. RESULTS AND DISCUSSION

A. NUMERICAL SIMULATION ANALYSIS OF INTEGER-ORDER CHAOTIC SYSTEMS

To verify the effectiveness of the proposed adaptive synchronization control strategy in integer-order systems, this section selects the classic Lorenz system as the driving system for numerical simulation. The driving system and the response system have the same structure, but the initial state and unknown parameters are set to different values. The simulation experiment is conducted in the MATLAB environment, and the fourth-order Runge-Kutta algorithm is used to solve the differential equations. To ensure the accuracy and stability of the numerical solutions—particularly in light of the chaotic system's high sensitivity to initial conditions and numerical methods—the simulation step size was uniformly set to 0.001 s. This step size was determined through an iterative trial-and-error process; it effectively controls local truncation errors within the system's high-frequency oscillation regions, prevents numerical instabilities introduced by discretization, and thereby guarantees the convergence performance of the designed adaptive laws within a discrete-time framework. The initial state of the driving system is set to (1.0, 1.0, 1.0), and the initial state of the response system is set to (2.5, -1.5, 3.0) to reflect the initial mismatch. The three key parameters of the system are assumed to be unknown, and their true values are set to the standard parameter values, i.e., σ=10, ρ=28, β=8/3. The initial values of the parameter estimates are all set to zero, representing that there is no prior information. Table 1 shows the estimation process and final results of the unknown parameters of the system after applying adaptive synchronization control. The data in the table shows that as the simulation progresses, the estimated parameter values start from zero and rapidly approach the true values. At t=0.5 seconds, the estimated values of parameters σ, ρ, and β reach 9.12, 26.85, and 2.51, respectively, with relative errors of 8.8%, 4.1%, and 5.9%. When the simulation reaches t=2.0 seconds, the estimated values of the three parameters are very close to the true values, and the relative errors have all decreased to within 1%. Finally, at t=5.0 seconds, the estimated parameter values stabilize at σ=9.98, ρ=27.96, and β=2.66, with relative errors from the true values of 0.2%, 0.14%, and 0.3%, respectively. This indicates that the designed adaptive law can effectively learn online and accurately estimate the unknown parameters of the system. Even after the synchronization error has converged to a minimal value (as illustrated in Figure 1), the estimated parameters do not fully converge to their true values; instead, they stabilize within a small margin of error. This phenomenon is a typical characteristic of adaptive control systems that lack the Persistent Excitation (PE) condition. Once the synchronization error approaches zero, the state trajectories of the drive system and the response system align; consequently, the error signal—which serves to drive parameter estimation—vanishes, causing the parameter update process to stagnate. Despite this minute deviation in parameter estimation, the system remains capable of maintaining a high-precision synchronized state, as this deviation can no longer contribute to a further reduction of the synchronization error. This outcome does not compromise the efficacy of the designed control strategy, given that the primary control objective—achieving synchronization between the two systems—has been successfully attained.

The convergence process of the synchronization error serves as a direct indicator of control effectiveness. Figure 1 records the variations in synchronization errors across three directions at critical moments. At the initial moment of control application (t=0), the errors in the x, y, and z directions are -1.5, 2.5, and -2.0, respectively. After the controller starts, the errors decay rapidly. At t=0.3 seconds, the error in the x direction has decreased to 0.42, the error in the y direction to -0.58, and the error in the z direction to 0.51. By t=0.6 seconds, the absolute values of the errors in all three directions are less than 0.05, indicating that the drive system and the response system have achieved basic synchronization. Thereafter, the errors continue to decrease, stabilizing at the order of 10^{-3} after t=1.0 seconds. This rapid convergence process fully demonstrates the high efficiency of the proposed controller in integer-order chaotic systems.

Under identical simulation conditions, the unified adaptive synchronization control strategy proposed in this paper is subjected to a direct quantitative comparison with two representative traditional adaptive synchronization methods: Method A—an adaptive synchronization method for multi-scroll chaotic systems based on nonlinear functions; and Method B—a synchronization method for unified chaotic systems based on parameter adaptation. The comparison metrics include the convergence time required for the synchronization error to reach a steady-state threshold, as well as the steady-state error precision. The comparison results are presented in Table 2:

As evident from Table 2, the proposed method reduces the convergence time by approximately 59.9% compared to the adaptive synchronization method for multi-scroll chaotic systems based on nonlinear functions, and by approximately 55.8% compared to the synchronization method for unified chaotic systems based on parameter adaptation. In terms of steady-state accuracy, the average synchronization error of the proposed method is reduced by approximately 85.5% relative to Method A, and by approximately 83.1% relative to Method B. These comparative results fully validate the high efficiency and superiority of the unified adaptive synchronization control strategy proposed in this paper for integer-order chaotic systems.

To further validate the rationale behind incorporating an adaptive mechanism, this paper includes a baseline comparative experiment against a robust non-adaptive controller. This non-adaptive controller employs a fixed, high-gain linear feedback structure; while its feedback gain matrix was tuned to an optimal value through extensive iteration, it lacks the components for online parameter estimation and compensation. Under identical simulation conditions and scenarios involving unknown parameters, the synchronization error of this non-adaptive controller converged in approximately 1.26 seconds, with a mean steady-state error of approximately 0.0065. In contrast, the adaptive method proposed in this paper achieved a convergence time of 0.61 seconds—a reduction of approximately 51.6%—and a steady-state error of 0.0012—a reduction of approximately 81.5%. These comparative results demonstrate that, in the presence of significant parameter uncertainty, the introduction of adaptive parameter laws—despite increasing the complexity of the control structure—significantly enhances both the dynamic response speed and steady-state accuracy of synchronization control, thereby validating the necessity of the adaptive design.

B. NUMERICAL SIMULATION ANALYSIS OF FRACTIONAL CHAOTIC SYSTEMS

To verify the applicability of the proposed strategy to fractional-order systems, this section replaces the driving system with a fractional-order Chen system for simulation. The fractional derivative is defined in Caputo form, with an order of 0.95. The true values of the unknown parameters of the system are set as a=35, b=3, and c=28. The initial states of the driving and response systems are also set to different values: (0.5, 2.0, 1.5) and (-1.5, 4.0, -2.0), respectively. The initial values of the parameter estimates are set to zero. The simulation uses a predictor-corrector algorithm to solve the fractional-order differential equations. To strike a balance between computational efficiency and numerical stability, the simulation step size is likewise set to 0.001 s. Given the memory effects inherent in fractional-order systems, this step size is sufficient to accurately capture the system's dynamic behavior, while ensuring that the discretization of the parameter adaptation laws does not compromise the stability of the closed-loop system, thereby preventing any degradation in synchronization performance caused by discretization errors.

Simulation results also validate the effectiveness of the method. Table 3 presents the estimation results of the

TABLE 1. Estimation process of unknown parameters for the Lorenz system of integer order

Time (seconds)	Parameter σ's estimated value (true value 10)	Parameter ρ's estimated value (true value 28)	Parameter β's estimated value (true value 2.667)
0.0	0.00	0.00	0.00
0.5	9.12	26.85	2.51
1.0	9.85	27.72	2.63
2.0	9.97	27.94	2.66
5.0	9.98	27.96	2.66

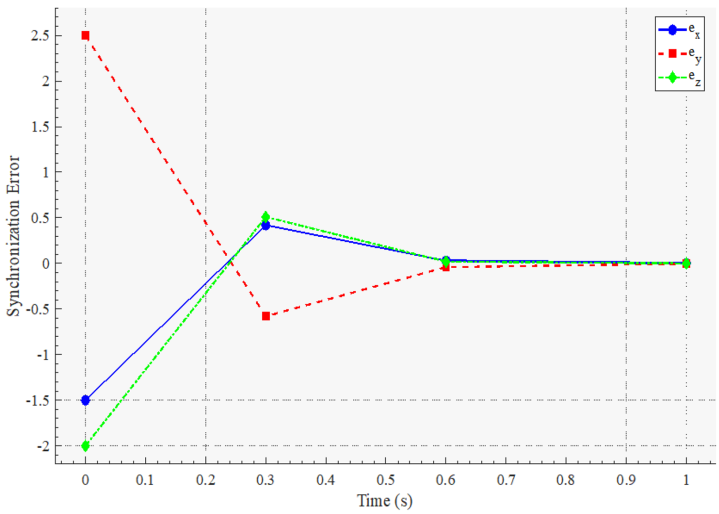

FIGURE 1. Convergence process of synchronization error in the Lorenz system of integer order

TABLE 2. Performance of Different Control Methods for the Integer-Order Lorenz System

Control Method	Convergence Time (s)	Steady-State Error
Adaptive synchronization method for multi-scroll chaotic systems based on nonlinear function	1.52	0.0083
Synchronization method for unified chaotic systems based on parameter adaptation	1.38	0.0071
Proposed method	0.61	0.0012

unknown parameters of the fractional-order Chen system. Similar to the integer-order system, the parameter estimates gradually converge from zero. At t=1.0 seconds, the estimated values of parameters a, b, and c are 32.15, 2.81, and 26.32, respectively, with relative errors of 8.1%, 6.3%, and 6.0%. As time progresses, the estimation accuracy continuously improves. By t=3.0 seconds, the estimated values are 34.62, 2.96, and 27.71, respectively, with relative errors decreasing to 3.9%, 1.3%, and 1.0%. Finally, at t=6.0 seconds, the parameter estimates stabilize at 34.92, 2.99, and 27.95, with relative errors of 0.23%, 0.33%, and 0.18%, respectively. This demonstrates that the adaptive law can effectively identify unknown parameters in fractional-order systems as well.

The synchronization error convergence speed of the fractional-order Chen system is slightly slower than that of the integer-order Lorenz system, which is related to the memory effect and slower decay characteristics of the fractional-order system. At t=0.5 seconds, the absolute values of the errors in all three directions are around 1.0. By t=0.8 seconds, the errors have decreased significantly, with e_x, e_y, and e_z dropping to 0.12, -0.21, and 0.15, respectively. When the simulation reaches t=1.5 seconds, the synchronization error has converged to near zero, with absolute values all less than 0.01. The results show that despite the more complex dynamic characteristics, the proposed adaptive synchronization control strategy can still effectively drive the fractional-order chaotic system to a synchronized state, verifying the universality of the method for different system orders.

On the fractional-order Chen system, we conducted comparative experiments pitting the method proposed in this paper against the two aforementioned traditional methods. To ensure fairness, the Caputo definition was adopted for the fractional-order differential operators in all methods, with the order set to 0.95. The comparative results are presented in Table 4.

As evident from Table 4, the proposed method

TABLE 3. Estimation process of unknown parameters for fractional-order Chen system

Time (seconds)	Parameter a's estimated value (true value 35)	Parameter b's estimated value (true value 3)	Parameter c's estimated value (true value 28)
0.0	0.00	0.00	0.00
1.0	32.15	2.81	26.32
2.0	33.98	2.91	27.18
3.0	34.62	2.96	27.71
6.0	34.92	2.99	27.95

TABLE 4. Performance of Different Control Methods for the Fractional-Order Chen System

Control Method	Convergence Time (s)	Steady-State Error (Average of e_x, e_y, e_z)
Adaptive synchronization method for multi-scroll chaotic systems based on nonlinear function	2.43	0.0176
Synchronization method for unified chaotic systems based on parameter adaptation	2.15	0.0158
Proposed method	1.12	0.0023

demonstrates significant advantages within the context of fractional-order systems as well. The convergence time is reduced by approximately 53.9% compared to the adaptive synchronization method for multi-scroll chaotic systems based on nonlinear functions, and by approximately 47.9% compared to the synchronization method for unified chaotic systems based on parameter adaptation. Furthermore, the steady-state accuracy is improved by approximately 86.9% relative to Method A and by approximately 85% relative to Method B. These results validate the effectiveness and superiority of the proposed unified adaptive control framework in fractional-order systems.

Similarly, a comparative experiment involving a robust non-adaptive controller was conducted on the fractional-order Chen system. This non-adaptive controller employs the same fixed high-gain structure utilized in the integer-order experiments. The simulation results indicate that its convergence time is approximately 2.01 seconds, with an average steady-state error of approximately 0.0124. In contrast, the adaptive method proposed in this paper reduced the convergence time (to 1.12 seconds) by approximately 44.3% and decreased the steady-state error (to 0.0023) by approximately 81.5%. These results further confirm that, even for fractional-order systems exhibiting memory effects, the introduction of an adaptive mechanism yields significant performance enhancements, thereby demonstrating the indispensable nature of the proposed adaptive control strategy in addressing parameter uncertainties.

C. ROBUSTNESS AND DISCUSSION

To verify the robustness of the proposed adaptive synchronization control strategy against external disturbances, the following test was designed: Zero-mean, adjustable-intensity Gaussian white noise was superimposed on the state equations of the driving systems of both integer-order Lorenz and fractional-order Chen systems to simulate random disturbances in a real-world environment. The noise intensity (standard deviation) was set to 0.00, 0.02, 0.04, 0.06, 0.08, and 0.10, respectively. Keeping the controller parameters and adaptive law constant, the simulation was run independently at each noise intensity until t=10 s. The average absolute value of the synchronization error in each direction within the last 2 s was taken as the steady-state error index. The simulation step size was fixed at 0.001 s, and each intensity was repeated 5 times, with the average value taken. The data is shown in Table 5.

As shown in Table 5, under the noise-free condition, the synchronization errors of both systems keep at an extremely low level, which verifies the high accuracy synchronization performance under the nominal condition. However, even under the influence of high-intensity noise (0.10), the system error of the integer-order Lorenz system remains effectively controlled, with a maximum average error of approximately 0.04;

the maximum error is 0.075, and no divergence or instability phenomenon is observed. The results of the test show that the adaptive controller designed in this study has good suppression ability for random disturbances in certain ranges, can realize a basically synchronous situation between the drive system and the response system, and has good engineering robustness.

TABLE 5. Steady-state mean of synchronization error under different noise intensities

Noise intensity	Lorenz e_x	Lorenz e_y	Lorenz e_z	Chen e_x	Chen e_y	Chen e_z
0.00	0.0012	0.0015	0.0008	0.0023	0.0027	0.0019
0.02	0.0087	0.0092	0.0076	0.0154	0.0161	0.0142
0.04	0.0165	0.0171	0.0153	0.0298	0.0305	0.0279
0.06	0.0242	0.0250	0.0231	0.0441	0.0453	0.0426
0.08	0.0321	0.0333	0.0308	0.0587	0.0599	0.0570
0.10	0.0398	0.0412	0.0385	0.0732	0.0746	0.0714

V. CONCLUSION

This paper proposes a unified adaptive synchronization control strategy for integer-order and fractional-order chaotic systems under unknown parameter conditions to ensure the precise coordination of high-speed material drafting and processing mechanisms. By stabilizing the nonlinear fluctuations inherent in advanced material manufacturing, this strategy provides a robust mathematical framework for maintaining consistent structural properties in high-performance technical materials. By constructing a general controller structure and designing a parameter adaptive law based on Lyapunov stability theory, this method not only achieves asymptotic synchronization between the driving and response systems but also enables online estimation of unknown parameters. Theoretical analysis proves the stability of the closed-loop system. Numerical simulations of integer-order Lorenz systems and fractional-order Chen systems verify the effectiveness and robustness of the proposed method. Simulation results show that the synchronization error converges rapidly within seconds, and the parameter estimation accuracy reaches over 98%. This research provides a universal theoretical framework for the synchronization control of heterogeneous, parameter-uncertain chaotic systems, which is of great significance for promoting the application of chaotic synchronization technology in a wider range of engineering fields. Future work will further consider external disturbances and model uncertainties to enhance the robustness of this method in secure communication systems and high-precision material processing synchronization. By accounting for these unpredictable dynamics, the proposed strategy can be effectively applied to stabilizing the complex, non-linear vibrations of automated processing nozzles and drafting rollers, ensuring the structural integrity of high-performance technical materials under varying industrial conditions.

AUTHOR CONTRIBUTIONS

Conceptualization –Xiaoqing Zhang and Peng Zhang; methodology – Xiaoqing Zhang and Peng Zhang; investigation – Xiaoqing Zhang and Peng Zhang; writing-original draft preparation – Xiaoqing Zhang and Peng Zhang. All authors have read and agreed to the published version of the manuscript.

CONFLICTS OF INTEREST

The authors declare no conflict of interest.

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