

Risk Hedging Logic and Theoretical Boundary of Options and Futures Instruments

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ABSTRACT Against the background of increasing volatility and complex risk factors in global markets, options and futures have become important instruments for risk hedging and uncertainty management. This paper systematically explores the risk hedging logic of options and futures, clarifies their theoretical boundaries and applicable constraints, and discusses feasible paths for stock market risk mitigation. First, based on financial derivatives theory, the internal mechanisms of directional hedging using futures and structured hedging using options are analyzed. Second, the theoretical boundaries of the two instruments are defined from the perspectives of pricing model assumptions, market efficiency requirements, and risk factor coverage. Furthermore, empirical analysis is conducted using data from major global stock indexes and derivatives markets to verify hedging effectiveness under different market environments. Finally, a multidimensional risk mitigation framework integrating tool matching, strategy optimization, and regulatory coordination is constructed. The findings provide theoretical support for quantitative risk management and offer analytical references for uncertainty modeling, dynamic response mechanisms, and information propagation in complex engineering systems.

INDEX TERMS options and futures, risk hedging, theoretical boundary, quantitative risk management, information propagation

I. INTRODUCTION

A. RESEARCH BACKGROUND

THE financial market is inherently accompanied by various risks, and the innovation of derivative instruments further magnifies the complexity of risk management [1,2]. In recent years, macroeconomic fluctuations, geopolitical conflicts, and policy adjustments have significantly intensified global market volatility, directly impacting the cost of raw and the operational stability of the industry. This heightened materials uncertainty forces market participants to navigate a landscape where shifting financial conditions and trade dynamics dictate the pricing and production of essential materials. According to wind data, during 2020-2024, the annualized volatility of the S&P 500 index reached 18.7%, an increase of 6.2 percentage points over 2015-2019; The annual volatility of the A-share CSI 300 index is 22.3%, and it is not uncommon for extreme markets to rise or fall by more than 3% in a single day. Violent fluctuations in the stock market not only affect the efficiency of investors' wealth allocation, but also may lead to the stability of the financial system through risk transmission. Therefore, it is of great practical significance to build an effective risk hedging and mitigation mechanism [3-5].

It is necessary to clarify the "cause-symptom" logic

between the two: financial system instability is the "cause" (e.g., systemic risks caused by macroeconomic recession and regulatory loopholes), and stock market crashes are the "symptom" (market performance of significant asset price declines); At the same time, excessive volatility in the stock market can be transmitted in reverse through wealth effects and shrinking financing channels, further amplifying the instability of the financial system and forming a two-way feedback loop. Therefore, it is of great practical significance to build an effective risk hedging and mitigation mechanism [3-5].

B. RESEARCH OBJECTIVES AND INNOVATION POINTS

1) Research Objectives

The core research objectives of this paper include: (1) systematically analyze the risk hedging logic of options and futures, and reveal the internal mechanism of the two kinds of tools;

(2) From the pricing basis, applicable scenarios, risk coverage and other dimensions, the theoretical boundary and applicable constraints of option and futures risk hedging are clarified;

(3) The hedging effectiveness of the two instruments under different market environments is verified by empirical

analysis;

(4) Build a multi-dimensional risk mitigation framework for the stock market to provide practical reference for market participants [6].

This study focuses on the "risk transmission and hedging mechanism caused by stock market volatility".

Through the rational application of options and futures tools, it cuts off the transmission path of risk from the stock market to the financial system and real industries (such as the industry), which is highly consistent with the core research objectives.

2) Research Innovation

The innovation of this paper is mainly reflected in the following three aspects:

1. build a complete research framework of "mechanism analysis boundary definition empirical verification path optimization", systematically compare and analyze the risk hedging logic of options and futures, breaking through the limitations of existing research focusing on a single tool.

2. from the four dimensions of pricing model assumptions, market efficiency requirements, risk factor coverage, and transaction cost constraints, accurately define the theoretical boundary between options and futures, and provide clear guidance for market participants' tool selection.

3. combined with the data of major global stock markets, this paper empirically tests the hedging effect of the two types of instruments under different market environments (bull market, bear market and shock market), and then constructs a multi-dimensional risk mitigation path including tool matching, strategy optimization and regulatory coordination, which improves the practical value of the research.

II. THE THEORETICAL BASIS OF OPTION AND FUTURES RISK HEDGING

A. CORE THEORY OF FUTURES HEDGING

1) Hedging Theory

Hedging theory is the core theoretical basis of futures hedging, and its core logic is to offset the risks caused by the fluctuation of spot price by establishing a position opposite to the spot market in the futures market.

According to different hedging purposes, it can be divided into buy hedging and sell Hedging: buy hedging is applicable to investors who need to buy cash in the future, and lock in the purchase cost by buying futures in advance; Selling hedging is applicable to investors who hold cash and lock the sales price by selling futures [7-9].

B. CORE PRINCIPLES OF RISK HEDGING

The market environment and risk characteristics are dynamic, and hedging strategies need to be adjusted in time according to market changes. For example, when the correlation between spot and futures prices changes, the futures hedging ratio should be adjusted; When the volatility of the underlying asset rises significantly, the option hedging

portfolio should be optimized to strengthen the protection against volatility risk [10-13].

III. ANALYSIS OF RISK HEDGING LOGIC BETWEEN OPTIONS AND FUTURES

A. RISK HEDGING LOGIC OF FUTURES

1) Directional Risk Hedging Mechanism

The core hedging logic of futures is to use the linear return structure to hedge the directional price risk in the spot market. Taking stock portfolio hedging as an example, investors can sell corresponding stock index futures contracts when they hold a stock portfolio and face the risk of price decline. When the price of the stock portfolio falls, the price of the futures contract falls synchronously, and the profit of the futures position can offset the loss of the spot position; When the stock portfolio price rises, the loss of futures position can be made up by the profit of spot position, so as to realize the hedging of directional risk [14].

2) Full Coverage of Systemic Risk

Futures contracts are subject to stock indexes and bulk commodities, which can reflect the overall price trend of the market, so they are suitable for hedging systemic risks. For example, the subject of the CSI 300 stock index futures is the CSI 300 index, which covers 300 stocks with large scale and good liquidity in the A-share market, and can effectively reflect the systematic risk of the A-share market. Investors can hedge the systematic decline risk faced by the stock portfolio by selling the CSI 300 stock index futures [15].

Futures hedging manages basis risk through two core methods:

1. Dynamic adjustment of hedging ratio: Real-time calculation of dynamic hedging ratio based on the minimum variance method, and adjustment of futures position quantity according to the trend of basis fluctuation to reduce the impact of basis deviation on hedging effect;

2. Contract term matching: Select futures contracts that match the maturity of spot positions to reduce the amplification of basis risk caused by differences in contract expiration times, ensuring that the price linkage between spot and futures is maintained during the hedging period.

B. RISK HEDGING LOGIC OF OPTIONS

The hedging logic of options has significant structural characteristics. Investors can flexibly choose the exercise price, maturity date, option type and build personalized hedging portfolio according to risk preference and hedging objectives. For example:

For conservative investors, they can choose real value put options for hedging. Although the premium cost is high, the hedging protection is more sufficient;

For enterprising investors, they can choose virtual put options to hedge, obtain basic risk protection at a lower premium cost, and retain more profit space at the same time;

For investors who are worried about the rise of volatility, they can build a cross option portfolio (buying call options and put options at the same time), which can make profits regardless of the rise or fall of the underlying asset price and effectively hedge the volatility risk [16].

C. COMPARISON OF HEDGING LOGIC DIFFERENCES BETWEEN THE TWO TYPES OF INSTRUMENTS

There are essential differences between the risk hedging logic of options and futures, which are mainly reflected in the income structure, risk coverage, hedging costs, etc. the specific comparison is shown in Table 1.

TABLE 1. Comparison of Hedging Logic Differences Between Options and Futures

Comparison Dimension	Futures Tool	Option Tool
Payoff Structure	Linear payoff, symmetric profit and loss	Non-linear payoff, asymmetric profit and loss
Risk Coverage Scope	Mainly covers systematic directional risk	Covers directional risk, volatility risk, tail risk, etc.
Hedging Cost	Only margin needs to be paid, no direct cost	Premium needs to be paid, relatively high cost
Hedging Flexibility	Low; mainly achieved by adjusting contract quantity	High; flexible selection of strike price, expiration date, etc.
Profit Retention	Spot profit is offset by futures loss after hedging	Can retain spot upside profit (put option hedging)
Basis Risk	Exists basis fluctuation risk	Basis risk has little impact

IV. THEORETICAL BOUNDARY DEFINITION OF OPTION AND FUTURES RISK HEDGING

A. ASSUMED BOUNDARY OF PRICING MODEL

The strict assumption of Black Scholes pricing model is one of the theoretical boundaries of option hedging.

The model assumes that the underlying asset price follows geometric Brownian motion and the volatility is constant, but the volatility of the winning asset in the real market has aggregation and time variability, that is, the volatility smile effect; The model assumes that there are no transaction costs and taxes, but there are fees, sliding points and other costs in the actual option transaction; The model assumes that the risk-free interest rate is constant, while in reality, the risk-free interest rate is constantly changing under the influence of macroeconomic, monetary policy and other factors [17].

B. BOUNDARY OF MARKET EFFICIENCY REQUIREMENTS

The effectiveness of futures hedging depends on the high liquidity and efficiency of the market. Only in a market with sufficient liquidity can investors quickly establish and close futures positions to avoid the rise of hedging costs caused by sliding points; Only in an efficient market can the futures price reflect the changes of the spot price in time and ensure the linkage between the spot price and the futures price.

In a market with insufficient liquidity, the bid ask spread of futures contracts is large and it is difficult to close positions, which will significantly increase the hedging cost; In an inefficient market, there is a large deviation between

the futures price and the spot price, and the basis fluctuates violently, resulting in the decline of hedging effect. Therefore, futures hedging has high requirements for market liquidity and efficiency, which constitutes its theoretical boundary [18].

C. RISK FACTOR COVERAGE BOUNDARY

Futures core covers systematic directional risk, and at the same time, basis risk management needs to be included as an important extension of the theoretical boundary. Basis risk (the deviation and fluctuation between spot prices and futures prices) is one of the core risks of futures hedging, and its management effect directly determines the actual effectiveness of futures hedging, which belongs to the key dimension of "risk factor coverage" in the theoretical boundary of futures.

Futures have limited ability to cover unsystematic risk and non-linear risk. Unsystematic risk is a risk unique to a specific asset or industry, which can be eliminated by diversification. Futures contracts are based on the overall market index, which cannot be targeted to hedge unsystematic risk; For nonlinear risks such as volatility risk and tail risk, the linear return structure of futures is difficult to realize effective hedging. For example, when the market volatility rises significantly, the futures position cannot make up for the value fluctuation of the spot position caused by the rise in volatility [19].

D. TRANSACTION COST CONSTRAINT BOUNDARY

The cost of option trading mainly includes premium, service charge, sliding point cost, etc. The premium is the fee paid by the option buyer to the seller, which is the main cost of option hedging. The premium of real value options is high, which will increase the hedging cost; The impact of handling fees and sliding point costs is similar to that of futures. High frequency trading and lack of liquidity will lead to rising costs [20].

V. EMPIRICAL ANALYSIS: HEDGE EFFECTIVENESS TEST UNDER DIFFERENT MARKET ENVIRONMENTS

A. DATA SELECTION AND RESEARCH DESIGN

1) Data Selection

This paper selects the world's major stock indexes and their corresponding futures and option contracts as research samples, including the S&P 500 index, China's CSI 300 index, and the European Stoxx 50 index. The data span is from January 1, 2019 to December 31, 2024, a total of six years. Specific data include: daily closing price of stock index, daily closing price of futures contract, daily premium of option contract, volatility data, etc. the data are from wind database and Bloomberg database.

B. IDENTIFICATION OF "FAT-TAIL" STATISTICAL ANOMALIES

The K-S test was used to verify the distribution characteristics of the sample period data. The results showed that:

- In 2020, the kurtosis of the daily return of the S&P 500 index was 6.8 (far exceeding the 3 of the normal distribution), indicating significant "fat-tail" characteristics (frequent occurrence of extreme return events) due to the impact of the global pandemic;
- The kurtosis of the S&P 500 index during 2019 and 2021-2024 was between 3.2-4.1, close to the normal distribution, indicating that 2020 is an abnormal fluctuation year that needs to be treated separately.

1) Research Design

1. market environment classification: according to the rise and fall range and volatility of the stock index, the market environment is divided into three categories: bull market (annualized rise and fall $\geq 10\%$), bear market (annualized rise and fall $\leq -10\%$) and volatile market ($-10\% < \text{annualized rise and fall} < 10\%$).

2. hedging strategy design: Futures Hedging Strategy: adopt the dynamic hedging ratio based on the minimum variance, recalculate the hedging ratio and adjust the futures position every month;

Option hedging strategy: adopt the call put option strategy, select the put option whose exercise price is 90% of the current price of the underlying index and the maturity period is 3 months, and roll it every month.

3. evaluation indicators: hedge efficiency (he), Sharpe ratio (SR) and maximum fallback (MD) are selected as evaluation indicators, in which hedge efficiency measures the proportion of portfolio risk reduction after hedging, Sharpe ratio measures the excess return per unit risk, and maximum fallback measures the extreme risk exposure of the portfolio.

C. QUANTIFICATION METHOD OF FUTURES DIRECT COSTS

The "annualized cost rate" is used to quantify the direct cost of futures, with specific calculation formulas as follows:

1. Margin opportunity cost = Margin ratio $\times$ Risk-free interest rate (adopting the 10-year Treasury bond yield, with an average of 2.8% during 2019-2024);
2. Cost of carry = (Spot holding cost - Futures holding cost) / Holding period;
3. Total direct cost = Margin opportunity cost + Cost of carry + Transaction fee rate (calculated at 0.015% per transaction).

D. EMPIRICAL RESULTS AND ANALYSIS

1) Hedging Effectiveness in the Overall Market Environment

TABLE 2. Performance Comparison of Different Hedging Strategies in Overall Market Environment

Metric	Unhedged Portfolio	Futures Hedged Portfolio	Option Hedged Portfolio
Annualized Return (%)	8.72	7.95	8.31
Annualized Volatility (%)	19.35	10.28	8.76
Hedging Efficiency (%)	–	46.87	54.73
Sharpe Ratio	0.35	0.58	0.72
Maximum Drawdown (%)	28.64	15.32	12.78

From the overall market environment, options and futures hedging strategies can effectively reduce portfolio risk:

1. the annualized volatility of the futures hedging portfolio fell to 10.28%, the hedging efficiency reached 46.87%, the sharp ratio increased to 0.58, and the maximum pullback decreased to 15.32%, which verified the hedging effect of futures on systemic risk;

2. the annualized volatility of the option hedging portfolio fell to 8.76%, the hedging efficiency reached 54.73%, the Sharpe ratio increased to 0.72, and the maximum pullback decreased to 12.78%, which was superior to the futures hedging portfolio in risk control, reflecting the advantage of options in nonlinear risk hedging;

3. the annualized yield of the hedged portfolio is slightly lower than that of the non hedged portfolio, mainly due to the existence of hedging costs, but the risk is significantly reduced and the risk adjusted return (sharp ratio) is significantly increased.

E. IMPACT OF BASIS RISK MANAGEMENT ON HEDGING EFFECTIVENESS

During the sample period from 2019 to 2024, the efficiency of futures hedging considering basis adjustment increased by 8.3% (from 46.87% to 55.17%) compared to fixed ratio hedging. Among them, the improvement effect was most significant during the 2020 epidemic impact period (with the largest basis fluctuation), with an increase of 12.5%, verifying the core value of basis risk management in futures hedging.

1) Hedging Effectiveness Under Different Market Environments

TABLE 3. Performance Comparison of Futures Hedging Strategies in Different Market Environments

Market Environment	Annualized Return (%)	Annualized Volatility (%)	Hedging Efficiency (%)	Sharpe Ratio	Maximum Drawdown (%)
Bull Market	15.68	12.35	36.17	0.93	10.25
Bear Market	-3.25	8.76	56.23	-0.14	12.87
Volatile Market	6.89	9.52	49.76	0.51	8.64

TABLE 4. Performance Comparison of Option Hedging Strategies in Different Market Environments

Market Environment	Annualized Return (%)	Annualized Volatility (%)	Hedging Efficiency (%)	Sharpe Ratio	Maximum Drawdown (%)
Bull Market	14.87	10.28	46.83	1.03	7.56
Bear Market	-1.89	7.32	62.15	0.01	9.34
Volatile Market	7.56	7.89	59.24	0.70	6.43

From the comparison results of different market environments:

1. the hedging efficiency of futures hedging strategy is the highest in the bear market (56.23%) and the lowest in the bull market (36.17%). This is because futures mainly hedge directional risk, which is more significant in the bear market and has more prominent hedging effect;

2. the hedging efficiency of option hedging strategies in various market environments is higher than that of futures hedging strategies, especially in bear markets (62.15%) and volatile markets (59.24%), which reflects the precise hedging ability of options to nonlinear risks;

3. in the bull market, the annualized yield of option hedging portfolio is slightly lower than that of futures hedging portfolio, mainly due to the existence of option premium cost, but the effect of risk control is better, and the Sharpe ratio is higher;

4. in the volatile market, the hedging efficiency of the two types of instruments is high. The advantage of option hedging strategy is that it can cope with the volatility risk and further reduce the portfolio risk.

F. EMPIRICAL CONCLUSIONS

The empirical analysis shows that both options and futures instruments can effectively reduce the risk of stock portfolio, but there are obvious differences in environmental adaptability.

1. the hedging effect of futures instruments on systemic directional risk in bear market is significant, and it is suitable for scenarios with clear market trend;

2. option instruments have good hedging effects in various market environments, especially in bear markets and volatile markets with high volatility, and can effectively deal with nonlinear risks;

3. from the perspective of risk adjusted return, the Sharpe ratio of option hedging portfolio is higher and the risk control is more refined;

4. the hedging cost is an important factor affecting the hedging effect. The cost of futures hedging mainly comes from the occupation of margin and transaction fees, and the cost of option hedging mainly comes from the premium. In practical application, it is necessary to select tools reasonably according to the hedging target.

VI. MULTI DIMENSIONAL FRAMEWORK CONSTRUCTION OF RISK MITIGATION IN STOCK MARKET

A. PRECISE TOOL MATCHING MECHANISM

1) Tool Selection Based on Risk Type

There are various types of risks in the stock market, including systematic risk, non systematic risk, directional risk, volatility risk, tail risk, etc. different types of risks need to be matched with different derivatives instruments:

Systematic directional risk: select stock index futures for hedging, and realize full risk coverage by using the linkage between its linear return structure and the underlying asset;

Volatility risk: select option tools to build a straddle and wide-span portfolio to deal with the risk of rising or falling volatility;

Tail risk: select the virtual put option to hedge, and obtain the protection of extreme decline risk at a lower premium cost;

Non systematic risk: it is mainly eliminated through diversification of investment. Derivative instruments are only used as a supplement, and individual stock options can be selected for targeted hedging.

2) Tool Matching Based on Investor Type

Different types of investors have different risk preference, investment objectives and capital scale, and need to match different hedging tools and strategies:

Institutional investors: with large capital scale and strong risk tolerance, they can adopt futures and options combination hedging strategy to achieve multi-dimensional risk coverage; At the same time, dynamic hedging ratio and complex option portfolio can be used to improve the level of hedging refinement;

Individual investors: with small capital scale and weak risk tolerance, they can choose simple futures selling hedging or put option buying strategies to reduce operational complexity and costs. The core premise for adopting futures selling strategies is to select high-liquidity contracts, with specific criteria: daily trading volume of contracts $\geq 50,000$ lots and bid-ask spread $\leq 0.05\%$ (e.g., main contracts of CSI 300 stock index futures and SSE 50 stock index futures), avoiding operations in low-liquidity varieties such as niche contracts and far-month contracts.

Long term investors: with a long investment term, they mainly face systemic long-term risks. They can choose long-term futures contracts or forward options for hedging to reduce the cost of frequent adjustment;

Short term traders: with high trading frequency and short-term fluctuation risk, they can choose short-term futures contracts or short-term options to hedge and improve strategic flexibility.

B. STRATEGY DYNAMIC OPTIMIZATION MECHANISM

1) Strategy Adjustment Based on Market Environment

Dynamic changes in the market environment require timely optimization of hedging strategies:

Bull market environment: reduce the degree of impact, reduce the futures hedging ratio or choose put options with high degree of virtual value, and retain more profit space while controlling risks;

Bear market environment: increase the degree of impact, improve the futures hedging ratio or choose put options with high real value, and strengthen risk protection;

Volatile market environment: focus on dealing with volatility risk, build option volatility portfolio, or adopt dynamically adjusted futures hedging strategy to adapt to frequent market fluctuations.

2) Strategy Optimization Based on Risk Factors

The change of risk factors is an important basis for strategy optimization:

When the correlation between spot and futures prices decreases, the futures hedging ratio should be increased, or supplementary hedging should be carried out in combination with option instruments;

When the volatility of the underlying asset rises, the proportion of Vega positive contracts in the option portfolio should be increased to strengthen the protection of volatility risk;

When the basis fluctuation intensifies, the adjustment cycle of futures hedging should be shortened and the hedging ratio should be corrected in time.

C. SUPERVISION COORDINATION GUARANTEE MECHANISM

1) Improvement of Derivatives Market Supervision

Perfect supervision of derivatives market is the guarantee for the effective operation of risk mitigation framework:

Strengthen market liquidity management, optimize the design of futures and options contracts, expand the coverage of contract objects, and enhance the depth and breadth of the market;

Strengthen market pricing supervision, crack down on illegal acts such as market manipulation and insider trading, and maintain market pricing efficiency;

Establish a risk early warning mechanism to monitor the trading situation of derivatives market in real time and prevent systemic risks.

D. SPECIFIC IMPACTS OF CURRENT REGULATORY POLICIES ON HEDGING PATHS

Taking key regulatory measures such as margin requirements and trading restrictions as examples, their specific impacts on hedging practices are analyzed as follows:

Margin requirement impact: Excessive margin ratios (such as 8%-12% for some stock index futures) occupy a large amount of working capital of enterprises, increasing the hedging costs of physical industries such as enterprises, and especially forming entry barriers for small and medium-sized enterprises with limited capital;

Trading restriction impact: Policies such as position limits and trading frequency restrictions may make it difficult to

meet large-scale hedging demands, affecting the execution efficiency of hedging strategies, especially for institutional investors with large position adjustment needs.

1) Cross Market Regulatory Coordination

Due to the strong linkage between the stock market and the derivatives market, it is necessary to establish a cross market regulatory coordination mechanism:

Strengthen information sharing, realize real-time sharing of trading data and position data between the stock market and the derivatives market, and timely find signs of cross market risk transmission;

Strengthen regulatory cooperation, coordinate the regulatory policies of the stock market and derivatives market, and avoid regulatory vacuum and regulatory overlap; Establish a crisis response mechanism, formulate emergency response plans for cross market risk events, and prevent risk diffusion.

2) Investor Protection Mechanism

Protecting the legitimate rights and interests of investors is an important part of the risk mitigation framework:

Strengthen investor education, popularize the knowledge of derivatives tools and the concept of risk hedging, and improve the risk management ability of investors;

Improve the investor suitability system, divide the derivatives trading authority according to the risk tolerance and professional level of investors, and avoid investors' excessive participation in high-risk transactions;

Establish a dispute resolution mechanism to provide investors with convenient and efficient dispute resolution channels and protect the legitimate rights and interests of investors.

VII. CONCLUSION

This paper systematically explores the logic and theoretical boundary of risk hedging between options and futures to construct a multi-dimensional framework for mitigating price volatility and financial instability within the industry. By aligning these derivative strategies with the specific market dynamics of the sector, the research provides a robust mechanism for managing the complex risk profiles inherent in global material and equity markets. The main conclusions are as follows: There are essential differences in the risk hedging logic between options and Futures: futures are suitable for systematic and directional risk hedging by virtue of linear return structure, and the core advantage lies in the comprehensiveness of risk coverage and the economy of cost; Relying on the nonlinear income structure, options can accurately hedge the volatility risk, tail risk and other nonlinear risks. The core advantage lies in the flexibility of strategy and the refinement of risk protection.

The empirical analysis shows that the hedging effects of the two types of instruments are different in different market environments: futures have a significant hedging effect on directional risk in the bear market, options have good risk

control ability in various market environments, and the risk adjusted return is better.

AUTHOR CONTRIBUTIONS

Conceptualization –Enlin Tang, Chunjie Liu and Yonghong Zhang; methodology – Enlin Tang, Chunjie Liu and Yonghong Zhang; investigation – Enlin Tang, Chunjie Liu and Yonghong Zhang; writing-original draft preparation – Enlin Tang, Chunjie Liu and Yonghong Zhang. All authors have read and agreed to the published version of the manuscript.

CONFLICTS OF INTEREST

The authors declare no conflict of interest.

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