Constrained Least Square Algorithm based Emitter Localization under NLOS Condition

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Abstract

The emitter localization is a significant problem in many fields such as target tracking, wireless communication, radar and many types of mobile application. Range difference of arrival (RDOA) is recently used as a location determination method which utilizes range difference of signal arrival between each receiver. In RDOA localization, there are two kinds of factors which decrease the accuracy of estimation: the nonline-of-sight (NLOS) noise and measurement one. In this paper, we apply Kalman filter and constrained least square (CLS) algorithm for compensating these noises. With the proposed two algorithms, we can confirm high accuracy for localization. A simulation demonstrates the performance of our proposed algorithm.

1. Introduction

The global positioning system (GPS) is the most commonly used localization method. Since GPS signal necessarily requires a satellite signal, however, it can be easily distributed by jamming. In a few years, many localization methods using distributed signal receiver such as time of arrival (TOA), angle of arrival (AOA), received signal strength (RSS) are actively researched. RDOA, which is the localization method using range difference of signal arrival, receive great attention recently. In contrast with other localization method, RDOA technique does not require time synchronization between receivers since this technique formulates the localization equation with using range difference data between each receiver and predefined reference receiver. In this paper, we use RDOA technique for localization. In two dimensional coordinates, each RDOA indicates a hyperbola between two receivers. With using at least three receivers for localization, more than two defined hyperbolas intersect at specific spot in an ideal case. Since these hyperbolic equations are nonlinear by some noises, however, these equations do not derive one specific solution. [1]

In RDOA based localization system, the NLOS noise and measurement one are major factors which cause precision decline of estimate. In order to mitigate the NLOS problem, Venkatraman [2] proposed the use of various tests including Skewness and Kurtosis test, Shapiro-Wilk test, and Anderson-Darling test to distinguish LOS and NLOS propagation. Seow [3] presented a comprehensive NLOS localization scheme and a least square estimator based on AOA and TOA.

In this paper, we propose Kalman filter based recognition algorithm to identify the data partition which contains the NLOS noise. Also we suggest CLS algorithm based localization method. CLS algorithm is the optimization method based on least square (LS) algorithm. This paper is organized as follows. In Section 2, the NLOS recognition method based on Kalman filter is determined. With using identified LOS RDOA data, we analyze CLS algorithm based location determination technique which derive the solution more precisely than standard LS algorithm in Section 3. A simulation data is presented in Section 4 to confirm the performance of proposed algorithm. Finally, we conclude this paper in Section 5.

2. Kalman filter based NLOS recognition algorithm

Before we derive location estimation of an emitter, we need to consider the NLOS noise which causes distance inaccuracy at specific receiver. In this section, we divide measured range data into some partitions depending on time interval and recognize NLOS noise contained partition in each receiver. In order to identify NLOS partition at each receiver, Kalman filter can be applied to the state of emitter location.

The state of an emitter can be determined as two dimensional coordinates as follows

$$X_{k+1} = AX_k + \Gamma w_k, \quad k = 1, \ldots, M \tag{1}$$

with

$$A = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}, \ \Gamma = \begin{bmatrix} 0 \\ \Delta t \end{bmatrix},$$

and $X_k = [p_x \ v_x]^T$ is the state vector of x-location at the time sample t_k. w_k is the driving noise vector with a covariance of $C = \sigma_d^2 I$. p_x and v_x are the location and velocity of x-direction, respectively. In this paper, there are two state vectors of x-location and y-location which can express the location and velocity of an emitter. $Y_k = [p_y \ v_y]^T$ is the state vector of y-location.

The measured location vector Z_k can be expressed as

$$Z_k = HX_k + u_k \tag{2}$$

where $H = [1 \quad 0]$ and u_k is the measurement noise vector with a covariance of which the covariance matrix is $R = \sigma_m^2 I$.

From equations (1) and (2), the update process of the Kalman filter is denoted as follows

$$\hat{X}_k^- = A\hat{X}_{k-1} \tag{3}$$

$$P_k^- = AP_{k-1}A^T + \Gamma C \Gamma^T \tag{4}$$

$$K_k = P_k^- H^T (HP_k^- H^T + R)^{-1} \tag{5}$$

$$\hat{X}_k = \hat{X}_k^- + K_k(Z_k - H\hat{X}_k^-) \tag{6}$$

$$P_k = P_k^- - K_k HP_k^-. \tag{7}$$

Equations (3) and (4) are the time update process. Equations (5), (6) and (7) are the measurement update process. P_k denotes the covariance matrix of a state vector X_k and Z_k is Kalman filter gain.

We compare measured range data with estimated one derived from Kalman filter for recognizing the NLOS contained partition. The measured range data $r_m(t_k)$ between an emitter and *m*-th receiver at the time sample t_k can be formulated as

$$r_m(t_k) = r_i(t_k) + \Delta r_m(t_k) + \Delta r_{nlos}(t_k) \tag{8}$$

where $r_i(t_k)$, $\Delta r_m(t_k)$ and $\Delta r_{nlos}(t_k)$ denote the real range, the measurement range error and the NLOS range error, respectively.

In this paper, we suppose that there is one receiver which always exists under LOS environment. The standard deviation of measured range data between an emitter and each receiver is compared with the range standard deviation of an LOS receiver. The standard deviation ($\hat{\sigma}_{nlos}$) of unknown range data can be expressed as

$$\hat{\sigma}_{nlos} = \sqrt{\frac{1}{M}\sum_{k=1}^{M}(r_m(t_k) - r_{Kalman}(t_k))^2} \tag{9}$$

where $r_{Kalman}(t_k)$ is the estimated range by using Kalman filter. M is the number of range data in each partition.

Similarly, the standard deviation under LOS environment σ_m can be obtained from equation (9). Then, we can recognize whether each measured range data partition of a specific receiver is under NLOS environment or not with a subsequent hypothesis test.

$$H_1(LOS\ condition):\ \hat{\sigma}_{nlos} < \varepsilon\sigma_m$$

$$H_2(NLOS\ condition):\ \hat{\sigma}_{nlos} \geq \varepsilon\sigma_m \tag{10}$$

The parameter ε is chosen experimentally. If the identified range partition is under LOS environment, we progress the localization with this LOS RDOA data in order to estimate precise location of an emitter.

3. Localization using CLS method

In this section, we formulate the location expression equation based on RDOA method and determine the localization method using CLS algorithm. Although we recognized NLOS RDOA data, however, the measured data includes some errors due to measurement noise. In order to compensate this measurement error, the CLS algorithm is applied to our localization method. The CLS algorithm is one of the mathematical optimization methods which minimize the objective function in accordance with some constraints. In this paper, Lagrange multiplier method based CLS algorithm is applied to localization problem. [4]

3.1. RDOA based emitter location formulation

In order to apply CLS algorithm, the localization equation has to be formulated using recognized RDOA data. RDOA data for emitter location formulation is the LOS data obtained from the proposed Kalman filter based recognition method. If we gather RDOA data from at least three location-known receivers, we can derive localization formula of an emitter. We represent the unknown location of an emitter as $\mathbf{ms} = \left[p_x,\ p_y\right]^T$ and the known location of receivers are expressed as $\mathbf{bs}_i = \left[x_i,\ y_i\right]^T$, $i = \{1,\ 2,\ \ldots,\ N\}$. The RDOA measurement value can be obtained as follows

$$\begin{aligned} r_i &= \|\mathbf{ms} - \mathbf{bs}_i\| = \sqrt{(p_x - x_i)^2 + (p_y - y_i)^2} \\ r_{i1} &= \sqrt{(p_x - x_i)^2 + (p_y - y_i)^2} \\ &\quad - \sqrt{(p_x - x_1)^2 + (p_y - y_1)^2} \end{aligned} \tag{11}$$

where r_i is the range between an emitter and *i*-th receiver. With given $N-1$ RDOA data, the location of an emitter can be acquired as following equation

$$\mathbf{Gu} = \mathbf{b} \tag{12}$$

with

$$\mathbf{G} = \begin{bmatrix} x_2 - x_1 & y_2 - y_1 & r_{21} \\ \vdots & \vdots & \vdots \\ x_N - x_1 & y_N - y_1 & r_{N1} \end{bmatrix},$$

$$\mathbf{b} = \frac{1}{2}\begin{bmatrix} (x_2 - x_1)^2 + (y_2 - y_1)^2 - r_{21}^2 \\ \vdots \\ (x_N - x_1)^2 + (y_N - y_1)^2 - r_{N1}^2 \end{bmatrix}$$

and the parameter $\mathbf{u} = \left[p_x,\ p_y,\ r_1\right]^T$ is the solution of formulated localization equation which consists of emitter location. We can derive the LS solution using pseudoinverse. Since the LS solution contains some error,

however, we have to apply the optimization method to this formulation.

3.2. CLS based localization algorithm

In this section, the error between estimated solution and true one as following equation has to be minimized.

$$J_e = (\mathbf{G}\hat{\mathbf{u}} - \mathbf{b})^T (\mathbf{G}\hat{\mathbf{u}} - \mathbf{b}) \tag{13}$$

In equation (13), the estimate of parameter vector $\mathbf{u}$ is denoted by $\hat{\mathbf{u}}$.

In this CLS problem, we solve the objective function subject to the constraint as the following equation.

$$\hat{\mathbf{u}}^T \mathbf{\Sigma} \hat{\mathbf{u}} = 0 \tag{14}$$

where $\mathbf{\Sigma} = diag(1,\ 1,\ -1)$. Equation (14) can be rewritten as $(\hat{p}_x - x_1)^2 + (\hat{p}_y - y_1)^2 = \hat{r}_1^2$, equivalently. This optimization problem which is represented as equations (13) and (14) can be solved by using Lagrange multiplier method. [4] The Lagrange function denotes as follows

$$L(\hat{\mathbf{u}}, \eta) = (\mathbf{G}\hat{\mathbf{u}} - \mathbf{b})^T (\mathbf{G}\hat{\mathbf{u}} - \mathbf{b}) \quad \eta \hat{\mathbf{u}}^T \mathbf{\Sigma} \hat{\mathbf{u}} \tag{15}$$

where η is a Lagrange multiplier. The solution of this localization problem is the value which satisfies the derivative of Lagrange function with respect to $\hat{u}$ as zero.

The estimate of $\hat{u}$ by using proposed CLS based localization algorithm is derived as follows

$$\hat{\mathbf{u}} = (\mathbf{G}^T \mathbf{G} + \eta \mathbf{\Sigma})^{-1} \mathbf{G}^T \mathbf{b} \tag{16}$$

with η which satisfies the following equation.

$$\sum_{i=1}^{3} \frac{\alpha_i \beta_i}{(\eta + \lambda_i)^2} = 0 \tag{17}$$

where

$$\mathbf{G}^T \mathbf{G} \mathbf{\Sigma} = \mathbf{Q} \mathbf{\Lambda} \mathbf{Q}^{-1} \tag{18}$$

$$\boldsymbol{\alpha} = \mathbf{Q}^T \mathbf{\Sigma} \mathbf{G}^T \mathbf{b} = \left[\alpha_1, \alpha_2, \alpha_3\right]^T$$

$$\boldsymbol{\beta} = \mathbf{Q}^{-1} \mathbf{G}^T \mathbf{b} = \left[\beta_1, \beta_2, \beta_3\right]^T$$

Equation (18) is the eigenvalue decomposition process and $\mathbf{\Lambda} = diag(\lambda_1, \lambda_2, \lambda_3)$ is a diagonal matrix whose diagonal elements are the corresponding eigenvalues. Equation (16) is the optimized solution which minimizes the equation (13). The first and second elements of estimated parameter vector $\hat{\mathbf{u}}$ are the x-location and y-location, respectively. We can estimate more precise location of an emitter than standard LS solution.

4. Simulation

In this section, the reliability of the proposed algorithm is confirmed through a simulation. Our simulation includes five receivers which are (0, 0) km, (0, 100) km, (100, 0) km, (100, 100) km and (50, 100) km, respectively. We divide 200 time samples by 4 partitions. The NLOS noise is assumed to follow the Gaussian distribution with a mean of 3 and a variance of 1.5. Also a mean and variance of measurement noise are supposed as 1 and 1.5, respectively. Figure 1 shows the root mean square error (RMSE) comparison between NLOS noise contained result using standard LS algorithm and NLOS noise compensated result using proposed algorithm. The performance of our proposed algorithm can be verified by figure 1.

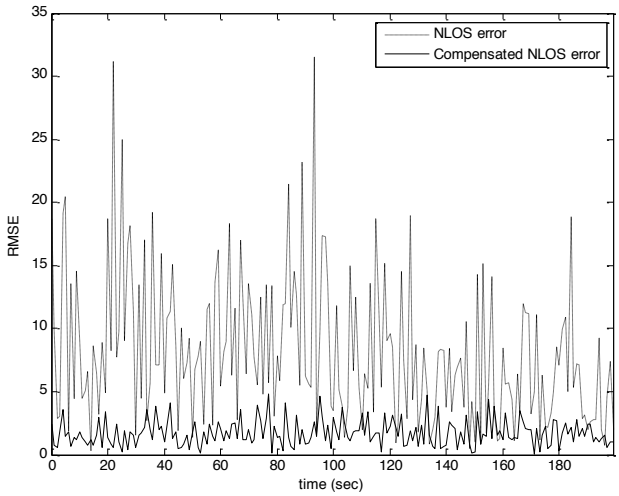

Figure 1: RMSE comparison between compensated result and uncompensated one

5. Conclusions

In this paper, we recognize the NLOS partition using Kalman filter. With the use of identified LOS data from each receiver, the Lagrange multiplier method based CLS algorithm is applied to RDOA based localization problem. We confirm a high performance of our proposed algorithm through a simulation. Although RDOA data contains NLOS noise and measurement one, we can derive precise location of emitter by using this algorithm.

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References

[1] K. Yang, J. An, X. Bu, and G. Sun, Constrained total least squares location algorithm using time difference of arrival measurements, *IEEE Trans. Veh. Technol.* 59: 1558−1562, 2010.

[2] S. Venkatraman and J. Caffery Jr., A statistical approach to non-line-of-sight BS identification, *Proc. 5th Int. Symp. Wirel. Pers. Multimedia Commun.*, Hawaii, USA, pp. 296−300, 2002.

[3] C. K. Seow, and S. Y Tan, Nonline-of-sight localization in multipath environments, *IEEE Trans. Mob. Comput.* 7: 647−660, 2008.

[4] K. W. Cheung, H. C. So, W. K. Ma and Y. T Chan, A constrained least squares approach to mobile positioning: algorithms and optimality, *EURASIP J. Appl. Signal Process. 2006*: 1−23, 2006.