

Thinned Smart Antenna of a Semi-circular Dipole Array for Massive MIMO Systems

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ABSTRACT Massive MIMO (multiple-input multiple-output) is a multi-user MIMO technology that can provide high-speed multimedia services in 5G wireless networks using sub-6 GHz and millimeter wave bands. The massive MIMO (MMIMO) installs array antennas in the base stations, using hundreds of transceivers with other RF modules. One of the drawbacks of the MMIMO system is its huge power consumption, and the beamforming network with RF modules for the large number of antennas is the main contributor to the power consumption. In this paper, a novel beamforming method is proposed for the low power consumption of an MMIMO system. The proposed thinned smart antenna (TSA) of a semi-circular array produces a secure beam towards the user's terminal with reduced interference. By thinning the antenna array, some of the antenna elements are kept off, resulting in less power consumption, while the array pattern remains the same as a fully populated array with a reduced side lobe level (SLL). The sub-6 GHz band of 5 GHz is used for the design of thinned array antennas. The genetic algorithm (GA) is used to determine the array sequence in thinning, and the adaptive signal processing algorithms least mean square (LMS), recursive least square (RLS), and sample matrix inversion (SMI) are used for the beamforming of the TSA, and the corresponding algorithms are GA-LMS, GA-RLS, and GA-SMI. The power saving of 40% to 55% is achieved using TSA. The maximum SLL reductions of 13 dB, 12 dB, and 14 dB are achieved for TSA using GA-LMS, GA-RLS, and GA-SMI algorithms, respectively.

INDEX TERMS Smart antenna, antenna array thinning, beamforming, adaptive signal processing, genetic algorithm, power consumption.

I. INTRODUCTION

THE key concept behind the deployment of MMIMO is to connect many terminals simultaneously with the same time-frequency resource using large antenna arrays in the base station. MIMO uses spatial diversity and spatial multiplexing to transmit separately and independently encoded data signals [1, 2]. In multi-user MIMO (MU-MIMO), different data signals to different users are sent using the same frequency and time to increase the network capacity [3, 4]. The MIMO is used in many modern wireless including Wi-Fi, and Long-Term Evolution (LTE). In MMIMO, advanced antenna technologies are used for beamforming to focus the signal in a predefined direction only by deploying larger antenna arrays [5, 6]. The MMIMO is a key building block of 5th generation new radio (5G NR) and provides multiple benefits to both network operators and end users [7, 8]. Spatial diversity, spatial multiplexing, and beamforming are the three main concepts used in MMIMO. Massive MIMO systems use tens, hundreds, or thousands of antennas in the antenna array at the base station for highly directive beam generation [9]. Due to the large number of antennas in the array, the power consumption in an MMIMO system is very high. There are several approaches generating highly

directive beams in an MMIMO system [10-16]. In [10], both dipole and microstrip antenna arrays are used in simulation for a massive MIMO system, and experimental results for microstrip arrays are reported, considering the signal arrival from different directions. The gain pattern variations are studied, considering mutual coupling in the antenna array. In [11], a 12-port hybrid 5G MMIMO antenna array is reported for different LTE bands and effect of user's hand is investigated. An overview of different types of beamforming techniques in 5G MMIMO system is described in [12]. The evolutionary algorithm with zero-forcing baseband beamforming for multi-user MMIMO is reported in [13] which have proposed hybrid beamforming system to improve the error profile. In [14], different types of MIMO array techniques to realize maximum system throughput is investigated which include planar array, circular array, and conformal array. Chebyshev tapering is proposed [15] for beamforming a multi-user MMIMO system to suppress the side lobes using a rectangular antenna array. The proposed hybrid beamforming is simpler compared to the conventional fully digital beamforming. In another report [16], a full-rank channel matrix is used for an MMIMO antenna system utilizing a fading emulator in the presence of a small number of virtually positioned

scatterers. A 64-element quasi-half-wavelength dipole circular antenna array is investigated using a 2D fading emulator. Recently, a review paper [17] was published where the design, development, and future directions of antenna technologies for 5G MMIMO are described.

Whatever the beamforming method for an MMIMO system, a large antenna array is required, which results in high power consumption. This particular issue of reducing of power consumption in an MMIMO system is not addressed in the literature. In this paper, the concept of a thinned smart antenna is proposed for the beamforming of the MMIMO system. This method of beamforming exploits the properties of the smart antenna (SA) and makes use of the property of a thinned antenna array, which reduces power consumption by keeping some of the antennas in the array off. An SA is an antenna array of any type of antenna that by using a digital signal processor, produces a highly secure radiation beam towards the user and a null towards the interferer [18-20]. The smart antenna identifies the direction of arrival (DOA) of the signal from the mobile and produces a retro-directive beam towards the user. The performance of the adaptive signal processing algorithm is the key factor in the performance of the smart antenna system. The most commonly used adaptive signal processing algorithms are LMS, RLS and SMI. In a thinned antenna array, by keeping some of the antenna elements strategically off, almost same beamwidth can be achieved with lower side lobe levels (SLLs) and reduced power consumption [21, 22]. The antenna array in which all the antennas are 'ON' is called a fully populated array. The ratio of the number of 'OFF' antennas to the number of 'ON' antennas is known as the thinning ratio. Practically no antenna element in an array is kept permanently 'OFF'; rather, according to requirements, the 'OFF' and 'ON' positions change. Matched loads or terminations, attached to the antenna connection are used to turn the antennas 'OFF'. The 'OFF' and 'ON' sequences in an array are determined either by statistical methods or by optimization methods. Due to their many advantages, optimization methods have become very popular for antenna array thinning. Generally, optimization methods like genetic algorithm (GA), particle swarm optimization (PSO), differential evolution (DE) etc. are used to obtain the optimal array sequence in a thinned array antenna [23-25]. Therefore, the proposed thinned smart antenna for MMIMO has all the properties of a smart antenna, and the thinning of the smart antenna makes the beam pattern have a reduced SLL and reduced power consumption. In this work, a semi-circular array of dipole antennas is considered to investigate the properties of the thinned smart antenna. There are some advantages to using circular, semi-circular, or elliptical arrays in a mobile tower [26, 27].

II. GENETIC ALGORITHM

In this work, GA is used for the optimization of antenna array to obtain the weight sequence of the dipole array, that is, the 'ON' and 'OFF' positions of the antenna elements in

the array. The GA is a simple and efficient global optimization method over continuous spaces [28-30]. The Genes of GA are the binary encodings of the parameters. An array of genes is a chromosome in the algorithm, and a chromosome has an assigned cost function that is to be evaluated. Then the selection process determines the parents and the selected parents reproduce by the GA operator, known as crossover. The crossover process is stopped after the completion of the new generation. During the mutation, the chromosomes are chosen randomly, and any randomly chosen bit is flipped from '0' to '1' and vice versa. After mutation, the old generation is replaced by the one with the best fitness. This method is repeated. After several iterations, all the chromosomes and their associated fitness become identical except for those that are mutated. Then the GA program stops [30]. In this program, double point crossover is used with roulette wheel selection method.

III. ADAPTIVE SIGNAL PROCESSING ALGORITHMS

After array thinning using GA, the array weight sequence is used for beamforming of SA using the LMS, RLS, and SMI algorithms, and the corresponding methods for thinned smart antennas are GA-LMS, GA-RLS and GA-SMI respectively. LMS is an adaptive filtering algorithm due to its simplicity [31, 32]. It is basically a stochastic gradient-based algorithm where the slope vector of the filter tap weight is utilized to meet the ideal Wiener solution. The filter weights of the adaptive filter are updated in every iteration using the formula [31, 32]

$$w(n+1) = w(n) + \mu e^*(n)x(n) \quad (1)$$

The algorithm minimizes the error $e(n)$ between the array output and the desired signal as

$$e(n) = d(n) - w^H(n)x(n) \quad (2)$$

Thus bound for the step-size parameter μ is given by

$$\mu < \frac{1}{2 \text{trace}[R_{xx}]} \quad (3)$$

The eigen values of the array correlation matrix determine the convergence speed of the LMS algorithm. In RLS algorithm this problem is solved by replacing the step size μ with a gain matrix. The least squares criterion is used in place of the statistical approach based on the MSE criterion to derive more quickly convergent adaptive filtering algorithms [31, 32]. The reference signal and correlation matrix information are the only requirements. In the RLS algorithm, the weight vectors are updated according to the equation

$$\bar{w}(k) = \bar{w}(k-1) + \bar{g}(k)[d^*(k) - \bar{x}^H(k)\bar{w}(k-1)] \quad (4)$$

Where, $\bar{g}(k) = \hat{R}_{xx}^{-1}(k)\bar{x}(k)$ is gain vector, $\hat{R}_{xx}$ is correlation matrix and given by

$$\hat{R}_{xx}(k) = \alpha \hat{R}_{xx}(k-1) + \bar{x}(k)\bar{x}^H(k) \quad (5)$$

Exponential weighting factor or forgetting factor ‘ α ’ is a positive constant, $0 \leq \alpha \leq 1$.

One of the disadvantages of LMS is that the algorithm needs more iteration for convergence to be satisfactory. To overcome this, sample matrix inversion (SMI) is used [31-33]. The sample matrix, which uses K-time samples, is a time-average approximation of the array correlation matrix. The temporal average estimate will match the real correlation matrix if the random process is ergodic in the correlation. For an ergodic random process, the time-average estimates will be same as the actual correlation matrix. SMI has a faster convergence rate because it uses direct inversion of the covariance matrix. The weights of the n -th block in the SMI algorithm are [17, 33]

$$w_{SMI}(n) = R_{xx}^{-1}(n)r(n) = [x_N(n) \ x_N^H(n)]^{-1}d^*(n)x_N(n) \quad (6)$$

where, the desired signal is

$$d(n) = [d(1+nK) \ d(2+nK) \ d(3+nK) \ \dots \ d(N+nK)] \quad (7)$$

In SMI algorithm, K is the block length.

IV. DESIGN OF THINNED SMART ANTENNA OF SEMI-CIRCULAR DIPOLE ARRAY

A uniform circular array (UCA) is a special arrangement for a uniform linear array (ULA). The uniform semi-circular array (USCA) is the half portion of a circular array and a typical semi-circular array for dipole antennas is shown in Figure 1. In this work, SA of semicircular uniform half-wave dipole antennas is considered. The dipoles are separated by uniform spacing of ‘ d ’.

For a dipole of length ‘ l ’, the far-zone electric field is [34]

$$E(\theta) = j\eta \frac{I_0 e^{-j\beta r}}{2\pi r} \left[\frac{\cos\left(\frac{\beta l}{2} \cos\theta\right) - \cos\left(\frac{\beta l}{2}\right)}{\sin\theta} \right] \quad (8)$$

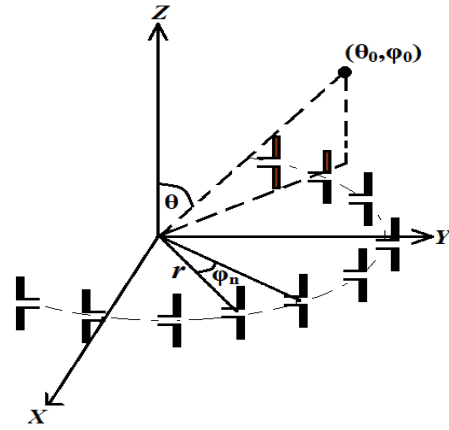


FIGURE 1. Uniform semi-circular array of dipole antennas.

Where, $\beta = 2\pi/\lambda$ is the propagation constant, I_0 is current amplitude, $\eta = 120\pi\Omega$ is the free space impedance and the observation point is at distance of r from the array. For N dipoles in the array, the total far-zone field is

$$E_{total} = E(\theta)AF(\theta) \quad (9)$$

where, $AF(\theta)$ is array factor (AF) for an array of isotropic elements for circular [34] and is given by

$$AF(\theta) = \sum_{n=1}^N I_0 e^{j[kr \sin\theta \cos(\varphi_0 - \varphi_n) + \beta_n]} \quad (10)$$

Where, N is the number of elements in the array, β_n is the phase excitation of n th elements, d_n is the spacing factor of the n th element, and kr and φ_n are given as

$$kr = \frac{2\pi r}{\lambda} = \sum_{i=1}^N d_i \quad (11)$$

$$\varphi_n = \frac{2\pi}{kr} \sum_{i=1}^n d_i \quad (12)$$

Where, λ is the wavelength, and it is assumed that all the antennas are fed by equal current I_0 . In case of semi circular we have taken half number of elements, so that the array factor becomes

$$AF(\theta) = \sum_{n=1}^{N/2} I_0 e^{j[kr \sin\theta \cos(\varphi_0 - \varphi_n) + \beta_n]} \quad (13)$$

The procedure for the beamforming of GA thinned smart antenna of dipole array is shown in Figure 2. First the dipole array is thinned by using GA optimization to obtain the ‘‘ON’’ and ‘‘OFF’’ sequence for the lowest SLL. Then this weighting sequence is used along with LMS, RLS and SMI to generate beam direction and null direction. The Eq. (9) for $E_{total}(\theta)$ is the cost function for thinned smart antenna.

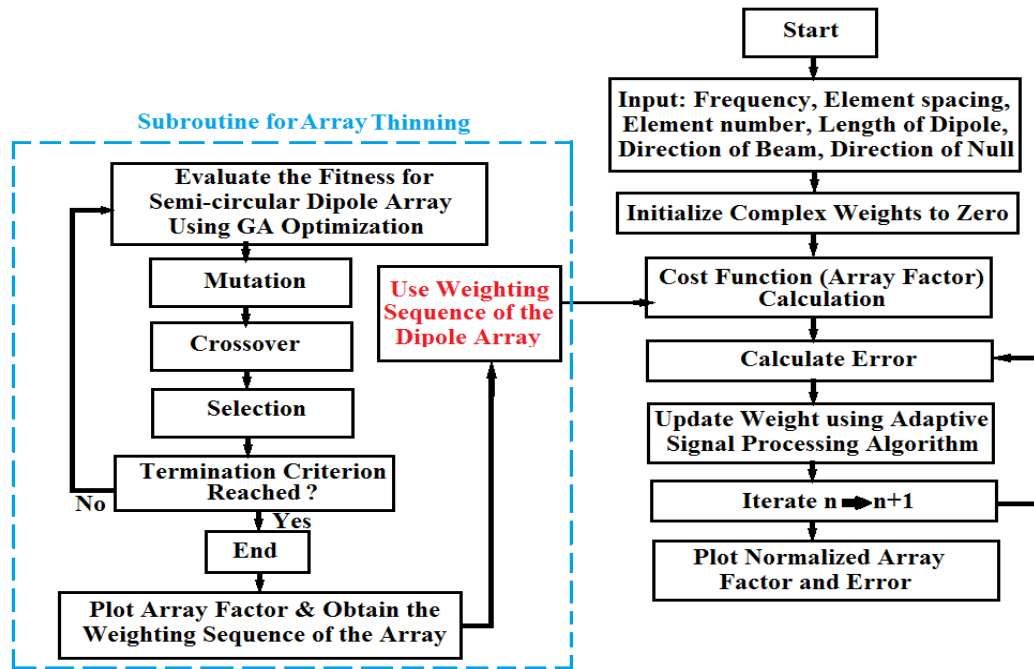


FIGURE 2. Method of beamforming of thinned smart antenna.

The parameters, used for the design of TSA, are given in Table I.

TABLE I. Simulation parameters

Number of dipole antennas in the array (N)	N=20, 31, 64
Inter-element spacing	0.5λ
Frequency	5 GHz (sub-6 GHz band)
Dipole length	$\lambda/2 = 0.03$ m
Signal-to-noise ratio (SNR)	20 dB
GA optimization parameters	Population size 40, Crossover percentage 0.8%, Mutation percentage 0.3%, Mutation rate 0.2
Value of step parameter (μ) in GA-LMS algorithm	0.002
Value of forgetting factor (α) in GA-RLS algorithm	0.9
Value of block length (K) in GA-SMI algorithm	800
Number of Iteration	800

For the beamforming of TSA of semi-circular dipole arrays, numbers of dipoles in the arrays are $N=20$, $N=31$ and $N=64$ are considered. The results for beamforming of thinned smart antenna using GA-LMS, for $N=20$ and $N=31$ are compared with the results for SA at different beam directions (BD) and null directions (ND), obtained without thinning and plotted in Figure 3, Figure 4 and Figure 5.

In Figure 3, 4, and 5, after comparison of the results of GA-LMS with that of LMS, it is found that the SLLs are reduced in GA-LMS by 11 dB for $N=20$, $BD=0^\circ$, 12 dB for $N=20$, $BD=20^\circ$, and 13 dB for $N=31$, $BD=0^\circ$.

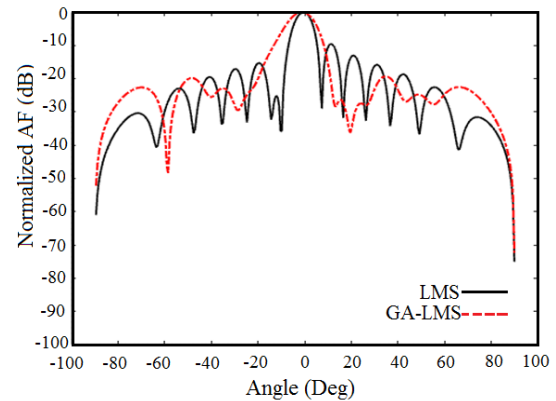


FIGURE 3. Array factor of TSA using GA-LMS for $N=20$ $BD=0^\circ$ $ND=10^\circ$.

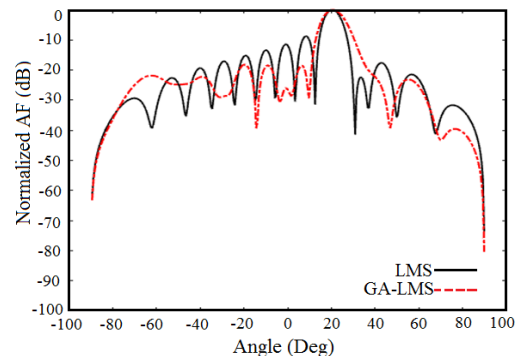


FIGURE 4. Array factor of TSA using GA-LMS for $N=20$ $BD=20^\circ$ $ND=10^\circ$.

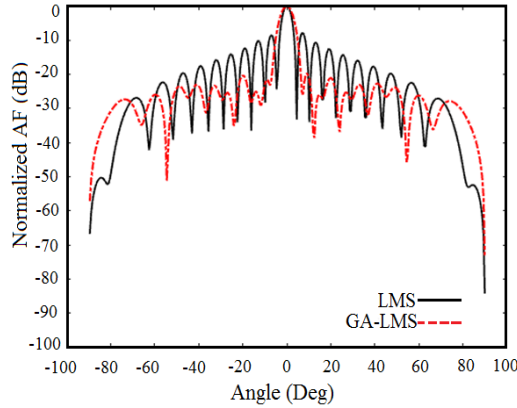


FIGURE 5. Array factor of TSA using GA-LMS for $N=31$ $BD=0^\circ$ $ND=10^\circ$.

The results for beamforming of thinned smart antenna using GA-RLS, for $N=20$, $N=31$, and $N=64$ are compared with the results for SA at different beam directions (BD) and null directions (ND), obtained without thinning and plotted in Figure 6, Figure 7, and Figure 8.

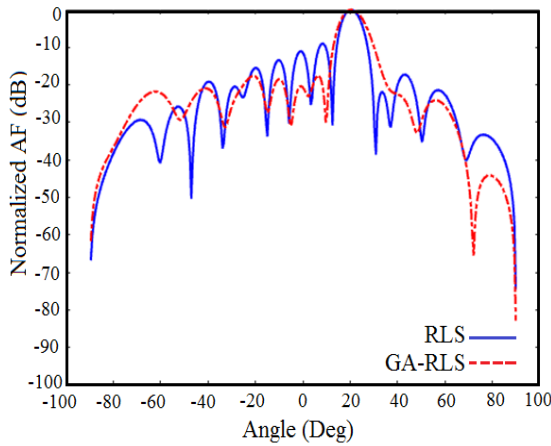


FIGURE 6. Array factor of TSA using GA-RLS for $N=20$ $BD=20^\circ$ $ND=10^\circ$.

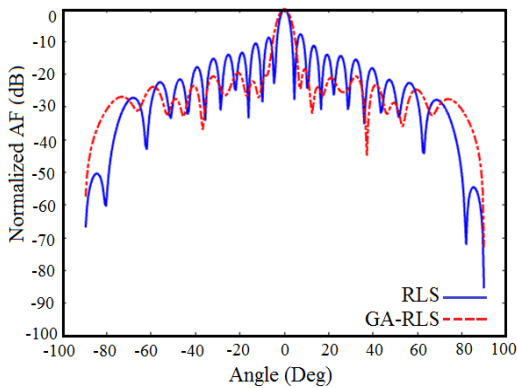


FIGURE 7. Array factor of TSA using GA-RLS for $N=31$ $BD=0^\circ$ $ND=10^\circ$.

In Figure 6, Figure 7, and Figure 8, after comparison of the results of GA-RLS with that of RLS, it is found that the SLLs are reduced in GA-RLS by 8 dB for $N=20$, 12 dB for $N=31$, and 14 dB for $N=64$.

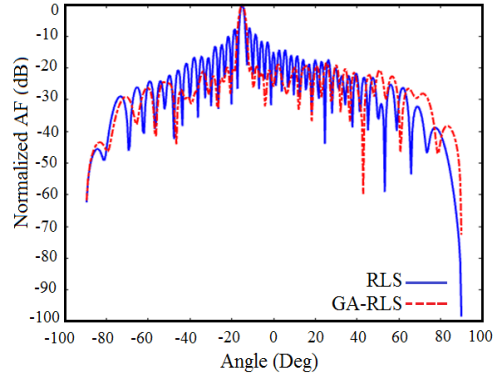


FIGURE 8. Array factor of TSA using GA-RLS for $N=64$ $BD=-15^\circ$ $ND=10^\circ$.

The results for beamforming of thinned smart antenna using GA-SMI, for $N=20$, $N=31$ and $N=64$ are compared with the results for SA at different beam directions (BD) and null directions (ND), obtained without thinning and plotted in Figure 9, Figure 10, and Figure 11.

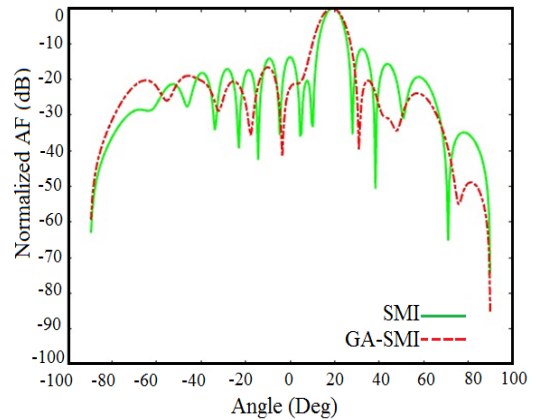


FIGURE 9. Array factor of TSA using GA-SMI for $N=20$ $BD=20^\circ$ $ND=10^\circ$.

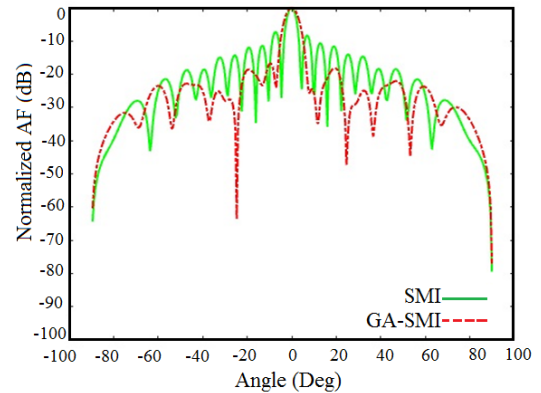


FIGURE 10. Array factor of TSA using GA-SMI for $N=31$ $BD=0^\circ$ $ND=10^\circ$.

In Figure 9, 10 and 11, after comparison of the results of GA-SMI with that of SMI, it is found that the SLLs are reduced in GA-SMI by 7 dB for $N=20$, 14 dB for $N=31$, and 13 dB for $N=64$.

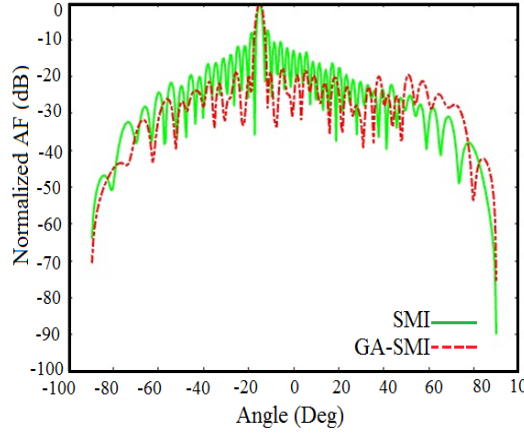


FIGURE 11. Array factor of TSA using GA-SMI for $N=64$ $BD=-15^\circ$ $ND=10^\circ$.

The profiles of best cost values for $N=20$, $N=31$, and $N=64$ are plotted in Figure 12. Here, the cost value is the magnitude of the array factor.

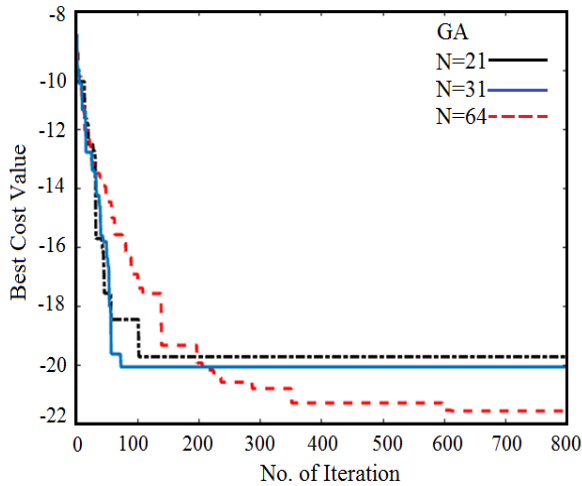


FIGURE 12. Best cost value for different numbers of dipole antennas in TSA.

The error graphs for $N=31$, obtained for thinned smart antenna using GA-LMS, GA-RLS and GA-SMI are shown in Figure 13.

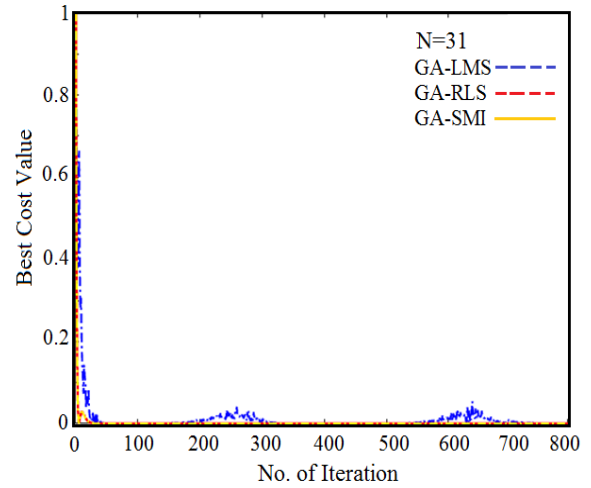


FIGURE 13. Error curves for TSA for $N=31$ for using different algorithms.

The performances of GA-LMS, GA-RLS, and GA-SMI are simulated for different numbers of dipole antennas and for different BD , and ND values. The performance of TSA are compared with SA and tabulated in Table II.

The 3dB beamwidths for the TSA almost remain identical like a SA without thinning which means directivity of the thinned smart antenna does not change so much. But SLL decreases appreciably by thinning of smart antenna. All the three algorithms GA-LMS, GA-RLS and GA-SMI provide desired BD and ND . The maximum power saving of 55% is achieved for TSA. The power savings are same for all the algorithms because array sequence is determined by the same GA optimization. But SLLs, an important parameter for interference in wireless communication, are different for all the three algorithms. The maximum SLL reductions of 13 dB, 12 dB and 14 dB are achieved for TSA using GA-LMS, GA-RLS, and GA-SMI algorithms, respectively compared to that for SA without thinning.

time is 6 minutes. The laptop used for simulation has following parameters: 11th Gen Intel(R) Core TM i3-1115G4 @ 3.00 GHz, 2901 MHz, 2 cores, 4 logical processors. The simulated results for the proposed algorithms could not be compared with other reports due to the unavailability of similar published papers. Investigations, presented in this paper show that both the reduction of SLL and power consumption is possible using thinned smart antenna without appreciable change of directivity of the smart antenna.

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