

## Resonance of High $T_c$ Superconducting Microstrip Patch in a Substrate-Superstrate Configuration

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### Abstract

The effect of a protecting dielectric superstrate on the resonance of a high  $T_c$  superconducting microstrip patch is investigated. The analysis approach is based on the spectral-domain method of moments in conjunction with the complex resistive boundary condition. The complex surface impedance of the superconducting thin film is determined using London's equation and the two-fluid model of Gorter and Casimir. Numerical results show that the resonant frequency of the high  $T_c$  superconducting rectangular patch decreases monotonically with increasing superstrate thickness, the decrease being greater for high permittivity loading.

### 1. Introduction

Microstrip devices have been used widely as microwave circuit elements such as transmission lines, filters, resonators, etc. The successful operation of, for example, the microstrip resonators seems to suggest that basically they are poor radiators [1]. Despite this weakness microstrip antennas have received much attention in the open literatures because of their many unique and attractive properties [2]: low in profile, light in weight, compact and conformable to both planar and non-planar surfaces [3], and easy to fabricate and to be integrated with solid-state devices. At low frequencies, the analysis for this structure can be done by using either the transmission model or a cavity model. However, for high frequency operation in the millimetre-wave range, the thin substrate approximation of low frequencies is not valid and a rigorous analysis is necessary for greater accuracy in the design of the element. The full-wave moment method has been applied extensively and is now a standard approach for analysis of patch antennas.

The discovery of a new class of copper-oxide superconductors which have transition temperatures above the boiling point of liquid nitrogen has resulted in much research aimed at exploiting the low microwave surface resistance of these materials. Passive planar superconductor circuits such as transmission lines, filters, and resonators have shown substantial improvement over identical circuits fabricated with gold, silver, or copper metallization [4-7]. The demonstration of practical microwave circuits utilizing

high-temperature superconductors has prompted investigations into how high-temperature superconductors may be used in antenna systems [8-11]. Although uses such as electrically short antennas and superdirective arrays have been proposed, the very high  $Q$  factors of such antennas make them rather impractical [8]. On the other hand, the benefits of using high-temperature superconductors in microwave and millimetre-wave microstrip array feed networks can be quite substantial owing to the reduced losses, which translates to an increase in the design of the antenna [10]. For a 35 GHz, 100-element linear array, Dinger [9] has shown that high-temperature superconductors may provide an 8 to 10 dB improvement in gain over a copper array.

Setting aside the topic of high-temperature superconductors, superstrate dielectric layers are often used to protect printed circuit antennas from environmental hazards, or may be naturally formed (e.g. ice layers) during flight or severe weather conditions. If the superstrate is spaced away from the patch antenna, a distance multiple of a half-wavelength, then the superstrate not only acts as a protecting layer, but also acts as a directive parasitic element, which increases the gain of the antenna [12]. When a superstrate is placed on top of a patch antenna, the resonant frequency of the antenna is shifted, and this shift may take the antenna out of its original operating frequency band. Therefore, an algorithm for the computation of the resonant frequencies of microstrip patches should be able to account for multilayered substrate effects. Theoretical researches on the effect of dielectric superstrate on the resonant frequency of a perfectly conducting patch are abundant [13]; however, there is no theoretical report on the effect of superstrate on the resonance of a high  $T_c$  superconducting rectangular microstrip antenna.

This paper presents a rigorous full-wave analysis of high  $T_c$  superconducting rectangular microstrip patch in a substrate-superstrate configuration. To include the effect of the superconductivity of the microstrip patch in the full-wave analysis, a complex surface impedance is considered. This impedance is determined by using London's equation and the two-fluid model of Gorter and Casimir [11]. Numerical results for the effect of the dielectric protecting superstrate on the resonant frequency of the high  $T_c$  superconducting rectangular microstrip antenna are given.

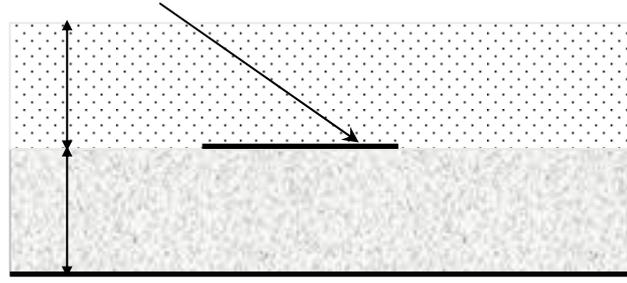


Figure 1: Geometrical Structure of a high  $T_c$  superconducting rectangular microstrip patch in a substrate-superstrate configuration.

## 2. Outline of the numerical procedure

The geometry under consideration is illustrated in Figure 1. A rectangular superconducting patch of thickness  $e$  is printed on a grounded dielectric slab of thickness  $d_1$ . The substrate is characterized by the free-space permeability  $\mu_0$  and a permittivity  $\varepsilon_0\varepsilon_{r1}$ . Above the radiating patch is the superstrate layer of thickness  $d_2$  with permeability  $\mu_0$  and a permittivity  $\varepsilon_0\varepsilon_{r2}$ . The superconducting film is characterized by a critical temperature  $T_c$ , a zero-temperature penetration depth  $\lambda_0$ , and a normal state conductivity  $\sigma_n$ . Following a mathematical reasoning similar to that shown in [13] for obtaining a relation among the surface electric field at the plane of the superconducting patch and the surface current on the patch in the spectral domain given by

$$\begin{bmatrix} \tilde{E}_x \\ \tilde{E}_y \end{bmatrix} = \begin{bmatrix} \Omega_{xx} & \Omega_{xy} \\ \Omega_{yx} & \Omega_{yy} \end{bmatrix} \cdot \begin{bmatrix} \tilde{J}_x \\ \tilde{J}_y \end{bmatrix}, \quad (1)$$

Where  $\Omega_{xx}$ ,  $\Omega_{xy}$ ,  $\Omega_{yx}$ , and  $\Omega_{yy}$  are the components of the spectral dyadic Green's function. The surface electric fields at the plane of the superconducting patch can be written as a superposition of an electric field in the patch and another out of the patch, this yields

$$\begin{cases} \tilde{E}_x = \tilde{E}_x^i + \tilde{E}_x^o \\ \tilde{E}_y = \tilde{E}_y^i + \tilde{E}_y^o \end{cases}, \quad (2)$$

The electric field in the superconducting rectangular patch is given by

$$\begin{cases} \tilde{E}_x^i = Z_s \tilde{J}_x \\ \tilde{E}_y^i = Z_s \tilde{J}_y \end{cases}, \quad (3)$$

where  $Z_s$  is the surface impedance of the superconducting patch. When the thickness of the superconducting patch is less than three times the penetration depth  $\lambda$  at a temperature  $T = 0$  K ( $\lambda_0$ ), the surface impedance can be expressed as in [14, Eq. (28)]. Substituting equations (2) and (3) in equation (1) yields

$$\begin{bmatrix} \tilde{E}_x^o \\ \tilde{E}_y^o \end{bmatrix} = \begin{bmatrix} \Omega_{xx} - Z_s & \Omega_{xy} \\ \Omega_{yx} & \Omega_{yy} - Z_s \end{bmatrix} \cdot \begin{bmatrix} \tilde{J}_x \\ \tilde{J}_y \end{bmatrix}, \quad (4)$$

Now, Galerkin moment method can be easily applied to equation (4) to obtain the complex resonant frequencies of the resonant modes of the high  $T_c$  superconducting rectangular microstrip patch in a substrate-superstrate configuration. Note that the resonant frequencies are defined as the real parts of the complex roots of the characteristic equation [15]. Muller's method is used for solving this characteristic equation.

## 3. Results

Although the full-wave analysis can give results for several resonant modes, only results for the  $TM_{01}$  mode are presented in the present work. This mode has a dominant current component in the  $y$  direction. In order to confirm the computation accuracy, our calculated resonant frequencies are compared with previously published experimental data [16]. The patch size is  $a \times b = 1.9$  cm x 2.29 cm. The substrate of thickness  $d = 1.59$  mm has a relative permittivity  $\varepsilon_{r1} = 2.32$ . Table 1 summarizes the measured and computed resonant frequencies for different superstrate materials and differences between numerical and experimental results of less than 1.17% are obtained. As a consequence, excellent agreement between theory and experiment is achieved.

Now, we study the influence of the superstrate thickness on the resonant frequency. A rectangular patch having a length  $a = 8$  mm and width  $b = 5$  mm is printed on a substrate of oxide of magnesium ( $\varepsilon_{r1} = 9.6$ ,  $d_1 = 0.4$  mm). For the superstrate we have considered three different materials. These materials are the arsenide of gallium ( $\varepsilon_{r2} = 6.6$ ), the oxide of magnesium ( $\varepsilon_{r2} = 9.6$ ) and the oxide of beryllium ( $\varepsilon_{r2} = 12.5$ ). The characteristics of the superconducting film are  $\sigma_n = 7.46 \cdot 10^6$  S/m,  $T = 77$  K,  $T_c = 89$  K,  $\lambda_0 = 180$  nm and  $e = 150$  nm. Figure 2 shows the resonant frequency versus the superstrate thickness. It is observed that when the superstrate thickness grows the resonant frequency decreases monotonically. The decrease being more important for superstrates with high relative permittivities.

Table 1: Comparison of our calculated resonant frequencies with experimental data [16] for perfectly conducting ground plane.

Superstrate			Resonant frequency (GHz)	
Dielectric	$\epsilon_{r2}$	$d_2$ ( $\mu\text{m}$ )	Measured	Our results
Custom High-K	10	3120	3.260	3.222
Plexiglas	2.6	3180	3.874	3.887
Mylar	3	64	4.070	4.108

#### 4. Conclusions

In this article, a rigorous full-wave analysis has been applied to investigate the effect of dielectric protecting superstrate on the resonant frequency of a high  $T_c$  superconducting rectangular microstrip patch. The superconductivity of the patch has been included in the theoretical formulation by means of the complex resistive boundary condition. To check the accuracy of the proposed method, our numerical results obtained for the case of perfectly conducting patch are compared with previously published data. This comparison has been done for various superstrate materials. In all cases, very good agreement between theory and experiment is achieved. Concerning the superstrate effect on the resonant frequency of the rectangular patch antenna, we have found that when the superstrate thickness grows the resonant frequency decreases monotonically. The decrease being more important for superstrates with high relative permittivities. This last behaviour agrees with that discovered theoretically for perfectly conducting patches [12].

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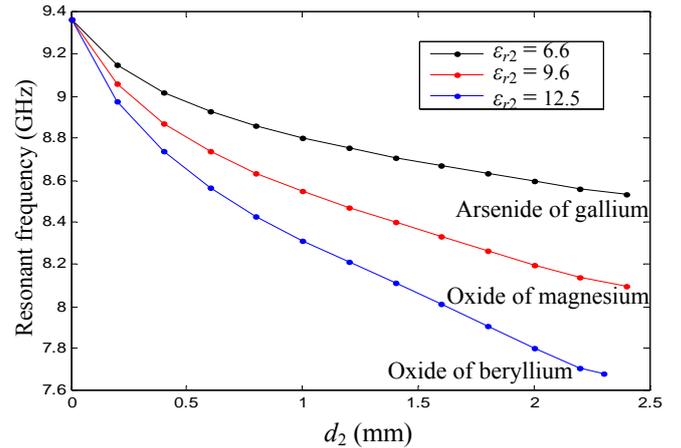


Figure 2: Resonant frequency of the superconducting patch versus the superstrate thickness.

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