Efficient CAD Model for the Analysis of High Tc Superconducting Circular Microstrip Antenna on Anisotropic Substrates

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Abstract

In this paper, an electromagnetic approach based on cavity model in conjunction with electromagnetic knowledge was developed. The cavity model combined with London's equations and the Gorter-Casimir two-fluid model has been improved to investigate the resonant characteristics of high Tc superconducting circular microstrip patch in the case where the patch is printed on uniaxially anisotropic substrate materials. Merits of our extended model include low computational cost and mathematical simplify. The numerical simulation of this modeling shows excellent agreement with experimental results available in the literature. Finally, numerical results for the dielectric anisotropic substrates effects on the operating frequencies for the case of the superconducting circular patch are also presented.

1. Introduction

The circular disc printed on a dielectric substrate backed by a perfectly conducting ground plane is used as an antenna as well as a resonator in microwave integrated circuits. Above the past years, microstrip resonators have been widely used over the range of microwave frequencies. In general, these structures are limited in its functional capabilities, but by proper design, the performances of these structures can be improved [1]. Since the discovery of the high critical temperature (Tc) superconducting materials which have critical temperatures above the boiling point of liquid nitrogen, the development of microwave application of high Tc superconductors has been extremely rapid, and numbers of highly sophisticated subsystem level modules have been generated [2]. Until now, it has been thought that the use of high Tc superconducting materials for microwave devices provided three important properties over their normal conducting counterparts. One is the lower surface resistance in high Tc superconducting thin films compared to normal conductors, corresponding to a higher quality factor [3] and improved performance in passive microwave devices. Also, because of lower losses in superconductors, the reduction in size is another advantage using high Tc superconducting thin films. The second advantage is the frequency independent penetration depth, unlike the common conductor. This means that dispersion introduced in superconducting devices will be negligible up to frequencies as high as hundreds of gigahertz [4]. The third advantage is that liquid nitrogen cheaper than liquid helium, as a refrigerant, can be used for cooling the superconducting devices because high Tc superconductors have Tc above the boiling point of liquid nitrogen, 77 K [2]. Therefore the researchers for high *Tc* superconducting microwave devices mainly have performed at the boiling point of liquid nitrogen, 77 K. However it could be needed to note that even below Tc the characteristics of high Tc superconducting microwave devices are varied with the temperature. Some dielectric substances exhibit anisotropy due to their natural crystal structures or as the result of their production processes [5]. Isotropic substances may also exhibit anisotropy at high frequencies. In the design of microwave integrated circuit components and microstrip patch antennas, anisotropic substances have been increasingly popular. Especially the effects of uniaxial type anisotropy have also been considered in some of the later studies [5-10] due to the availability of this kind of substances such as Sapphire, Magnesium fluoride and Epsilam-10. Formulations of those studies are lengthy and complicated even in single substrate case. In a previously presented study [6-8], we have shown that circular microstrip antenna with the properly selected uniaxial anisotropic substrate is more advantageous the isotropic one by exhibiting wider bandwidth characteristic with different resonant frequencies. Several computational methods have been carried for computing the resonant characteristics of superconducting circular microstrip patch antenna by using the spectral domain analysis and other full-wave analysis methods [6, 9]. With the increasing complexity of geometry and material property, designing these antennas requires more and more dedicated and sophisticated computer-aided-design (CAD) tools to predict the characteristics. These commercial design packages use computer-intensive numerical methods such as Finite Element Method (FEM), Method of Moment (MoM), Finite Difference Time Domain (FDTD) method, etc. These techniques require high computational resources and also take lots of computation time [11-12]. The computer-aided design (CAD) oriented conformal mapping, transmission line, and cavity model is ideal for design purpose because it involves less mathematical steps and less computational time. It is also easy to implement, provide closed form expressions [11-12]. These models have allowed assessing, with a good approximation, the different characteristics of planar structures made in microstrip technology [13]. Among them, fewer researchers [14-15], have investigated the effect of the anisotropic substrate on the resonant

characteristics of conventional conducting microstrip antennas. However, they have not studied thoroughly the effect of the superconducting patch on the resonant features of the antenna printed on uniaxially anisotropic substrates.
In this paper, we modify the cavity method for the analysis of circular disc microstrip antenna, in such a way that the method can treat the case of high *Tc* superconducting circular disc as well as the anisotropic substrate. The main advantage of this approach lies in its mathematical simplicity and low computation cost which is faster than the numerical methods and commercially available softwares.

2. Antenna design

The patch is assumed to be located on a grounded dielectric slab of infinite extent, and the ground plane is assumed to be a perfect electric conductor. The superconducting circular disc patch with radius *a* is embedded in a single uniaxially anisotropic substrate (ε_x , ε_z), which has a uniform thickness of *h* (Figure 1).

2.1. Resonant Frequency

The resonant frequency of the circular patch can be found analytically from the dimensions of the cavity comprised of the disk and ground plane using the cavity model. The discussion below follows the work of [16]. Assuming the cavity to be free of current sources, the Helmholtz equation is $\nabla^2 E + k_{nm}{}^2 = 0$, where k_{nm} is the propagation constant, $k_{nm} = 2\pi f_{nm}(\mu\varepsilon)^{1/2}$. Applying the boundary condition of the perfect magnetic wall, the electric field is found to be

$$E_z = E_0 J_n(k_{nm} r)\cos n\phi \tag{1}$$

In this equation, J_n is Bessel functions of order *n*. By the assumptions of the cavity model, the remaining components of the electric field, E_r and E_ϕ are assumed to be zero. The H-fields can be found by a simple application of Maxwell's curl equation and integration over time,

$$\overline{H} = \frac{j}{\mu\omega} \nabla \times \overline{E} \tag{2}$$

So that

$$H_r = \frac{j}{\mu\omega r}\frac{\partial E_z}{\partial \phi} = -\frac{jn}{\mu\omega r} E_0 J_n(k_{nm} r)\sin n\phi \tag{3-a}$$

and

$$H_\phi = -\frac{j}{\mu\omega}\frac{\partial E_z}{\partial r} = -\frac{jk_{nm}}{\mu\omega} E_0 J_n(k r)\cos n\phi \tag{3-b}$$

At the boundary of the disk the radial component of the surface current,

$$K = \hat{n} \times H \tag{4}$$

Must go to zero, which implies

$$J_n'(k_{nm} a) = 0 \tag{5}$$

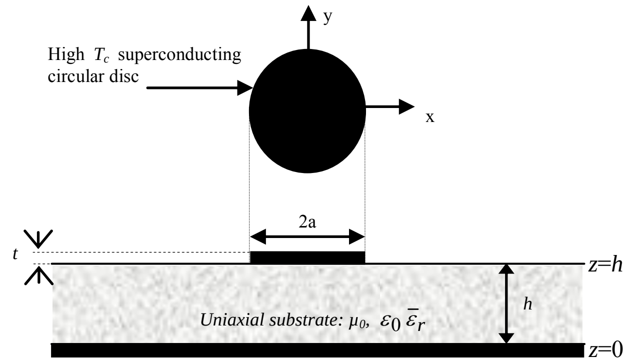

Figure 1: Geometrical structure of a high *Tc* superconducting circular disc microstrip antenna.

This is the condition for resonance. The integer *m* represents the *m*th zero of Eqn. (5). Thus, the resonant frequency is then given by

$$f_{mn} = \frac{X_{nm}}{2\pi a\sqrt{\varepsilon_r}} \tag{6}$$

Where ε_r is the permittivity of the dielectric inside the cavity and X_{nm} are the roots of $J'_n(x) = 0$ For a patch antenna, *a* is the radius of the patch and ε_r is the permittivity of the substrate. The first root of this equation corresponds to the lowest order mode (TM_{11}) and is X_{11}=1.8412.

This expression predicts frequencies substantially higher than measured frequencies because the model does not account for the fringing fields. Several expressions for a modified, "effective" radius have been proposed to account for the fringing fields. The most common expression is the one given by [17-18],

$$a_e = \left[1 + \frac{2h}{\pi a\,\varepsilon_r}\left(\ln\frac{\pi a}{2h} + 1.7726\right)\right]^{1/2} \tag{7-a}$$

The following patch radius expression can also be obtained from the static fringe capacitance given in [17],

$$a_e = a\left[1 + \frac{2h}{\pi a\,\varepsilon_r}\left(\begin{array}{l}\ln\frac{a}{2h} + (1.41\varepsilon_r + 1.77) \\ +\frac{h}{a}(0.268\varepsilon_r + 1.65)\end{array}\right)\right]^{1/2} \tag{7-b}$$

Using electromagnetic knowledge, the thickness h and the relative permittivities ε_x and ε_z of the uniaxially anisotropic substrate are replaced by effective parameters using the following equations [10],

$$\varepsilon_{re} = \varepsilon_z \tag{8-a}$$

$$h_e = h\sqrt{\frac{\varepsilon_x}{\varepsilon_z}} \quad (8\text{-b})$$

2.2. HTS Antenna's Resonant Frequency

The abrupt change in the high temperature superconducting (HTS) antenna's resonant frequency at temperatures near *T,* can be attributed to a modification of the magnetic penetration depth of the YBCO (YBa2Cu3O7). To model the resonant frequency of the HTS antenna, it is necessary to consider the effective dielectric constant, $\varepsilon_{r,eff}$, given by [19]

$$\varepsilon_{r,eff} = \varepsilon_{re}\left[1 + \frac{\lambda_f}{h}\coth\left(\frac{t}{\lambda_f}\right)\right] \quad (9)$$

For an HTS film with thickness *t* and the permittivity equivalent $\varepsilon_{re.}$ For a homogeneous superconductor, the temperature dependence of the magnetic penetration depth λ_f, can be modeled by the *Gorter-Casimir* two-fluid model as [19]

$$\lambda_f = \lambda_{f0}\left[1 - \left(\frac{T}{T_c}\right)^4\right]^{-1/2} \quad (10)$$

Where T is the temperature and T_c is the transition temperature of the superconductor film.

3. Numerical Results and Discussion

This section describes improvements and simulations performed on the cavity model, for superconducting circular microstrip antenna printed on uniaxially anisotropic substrates.

Firstly, to confirm the computation accuracy of the method described in the previous section, Table 1 shows the calculated resonant frequencies for different values of patch radius (a) printed on the isotropic substrate. These results are compared with theoretical and experimental data, which have been recently suggested in [20-21]. Note that the agreement between our computed results and the experimental results is excellent.

Table 1: Theoretical and experimental values of the resonant frequency for the fundamental mode of circular microstrip antennas on the isotropic substrate for conducting patch, h=0.787*mm*, ε_r= 2.2.

A / h	*Measured (GHz) [20]*	*Computed (GHz)*	
		[21]	Present
36.213	2.023	2.042	2.017
26.683	2.723	2.75	2.713
50.825	1.446	1.462	1.448

We now study the effect of anisotropy on the substrate, on the resonant frequency of perfect conducting circular microstrip patch antenna.

Figure 2 depicts the variation of the dominant mode of the resonant frequency with the change of perfect conducting patch radius (*a*). The antenna parameters for this study are h = 1.27 mm. The computed resonant frequency in Verma [14] for various anisotropic substrates is also shown to agree very well with the values computed using cavity model (6-9). The results of this comparison indicate that the present model shows excellent agreements with those of Verma [14] for all values of patch radius.

The circular microstrip antenna using high *Tc* superconducting thin film (YBa2Cu3O7) fabricated on $LaAlO_3$ substrate ($\varepsilon_r = 23.81$) of thickness ($h = 254\mu m$) and having a radius ($a = 610\mu m$) of thickness ($t = 330nm$), with penetration depth ($\lambda_0 = 1400nm$). The analysis is carried over the temperature range 20 K to 0.95 *Tc*, where *Tc* is the transition temperature (*Tc*=84.5 K) of the HTS film. Note that, the variation of the permittivity of the lanthanum aluminate substrate with the change of the temperature, as indicated by the experiment of Richard et al. [22], is taken into account in the present subsection. Results of our modified cavity model show better agreement with the experimental data (see figure 3), the error in the resonant frequency (less than 3%) is well within the tolerances of the substrate's material parameters. The difference between the measured and modeled resonant frequencies of the antennas is attributable to variations in the dielectric height between samples and also to the fabrication process; overreaching of the HTS antenna caused its radius to be $600\mu m$ instead of the desired $610\mu m$ [22].

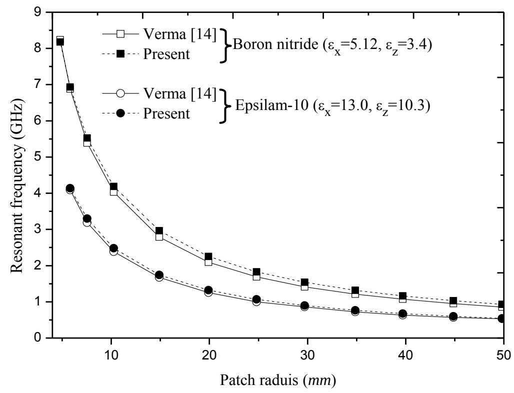

Figure 2: Resonant frequency of the dominant mode (TM^{11}) versus the patch radius (a), for two different anisotropic substrates materials.

The resonant frequencies of superconducting circular microstrip antenna as a function of Operating temperature for several anisotropic substrates is shown in Figure 4. The circular superconducting film of radius $a = 1700\mu m$ is of YBCO (YBa2Cu3O7) material characterized by: $\lambda 0 = 120nm$, $Tc = 84.5$K, $t = 250nm$, printed on different anisotropic substrates of thickness $h = 200\mu m$. From the results of figure 4, it can be observed that increasing the temperature will decrease the resonant frequency. This reduction is significant for temperatures near the transition temperature *Tc*. The effects of operating temperature, on the resonant frequency of a high-temperature superconducting circular patch printed on a different anisotropic substrate, excited in the TM^{11} mode, is shown in Figures 4. The variations of the resonant frequency are due to the uniaxial anisotropy substrate decrease gradually with the increase in the temperature.

This reduction becomes more significant for the values of temperature close to the critical temperature (*Tc*). These behaviors agree very well with those reported elsewhere in [7]. Note that the abrupt change in the resonant frequency at temperatures near Tc can be attributed to a modification of the magnetic penetration depth of the YBCO [23]. It also observed that the resonant frequencies obtained when the superconducting patch is printed on PTFE are higher than those obtained when the superconducting patch is printed on the Boron nitride or Quartz because the effective relative permittivity of the PTFE is lower than the one of both anisotropic materials Boron nitride and Quartz.

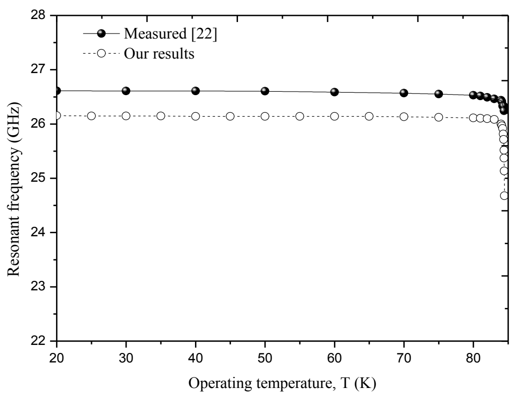

Figure 3: Resonant frequency for a circular microstrip superconducting patch antenna as a function of the operating temperature; $\varepsilon_r = 23.81$, $h = 254\mu m$, $a = 610\mu m$, $t = 330nm$, $\lambda_0 = 140nm$, $Tc = 84.5K$.

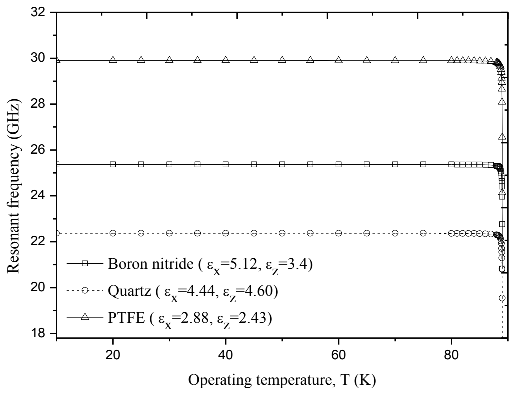

Figure 4: Resonant frequency versus operating temperature of superconducting circular microstrip patch, for three anisotropic substrates.

Figure5, show the influence of the operating temperature on the effective dielectric constant of superconducting circular microstrip patch, for three anisotropic substrates: Boron nitride ($\varepsilon_x = 5.12, \varepsilon_z = 3.4$), Quartz ($\varepsilon_x = 4.44, \varepsilon_z = 4.60$), and PTFE ($\varepsilon_x = 2.88, \varepsilon_z = 2.43$). The circular superconducting patch of the radius $a = 1700\mu m$ is of YBCO ($YBa_2Cu_3O_7$) material characterized by: $\lambda_0 = 120nm$, $Tc = 84.5K$, $t = 250nm$, printed on different anisotropic substrates of thickness $h = 200\mu m$. The effective permittivity of the substrate increases slowly with increasing operating temperature, while that of HTS patch is almost constant for temperature below *Tc*. The effective permittivity begins to growth and eventually become high near *Tc* because near *Tc*, the HTS materials start behaving like the common metals. Thus it can be concluded that the effect of uniaxial electric anisotropy on the resonant frequency of an HTS microstrip patch antenna cannot be ignored and must be taken into account in the design stage.

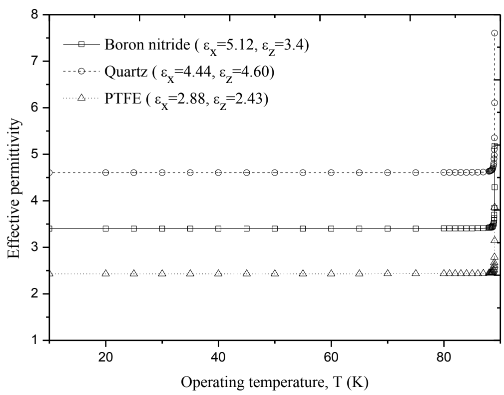

Figure 5: Resonant frequency versus operating temperature of superconducting circular microstrip patch, for three anisotropic substrates, (parameters as in Fig. 4).

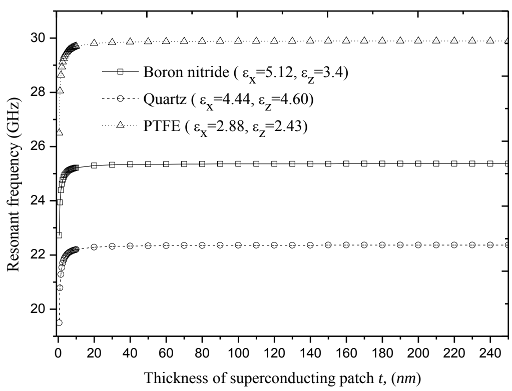

Figure 6: Resonant frequency versus thickness of superconducting patch, for three anisotropic substrates.

Figures 6 illustrate the variation of the resonant frequency versus the thickness of HTS circular microstrip patch using the same anisotropic substrates using in figure 5 ($T = 77K$). It can be seen that as the thickness of superconductor patch grows, the resonant frequency increases quickly until the thickness t reaches λ_0 (penetration depth). After this value, increasing the superconducting thickness will increase slowly the resonance frequency.

4. Conclusion

An efficient model based on cavity model in conjunction with electromagnetic knowledge has been described for computation of resonant frequencies of an HTS circular microstrip patch antenna on anisotropic substrates. The resonant frequencies calculated by using the present technique have been compared with those previously measured and available in the various literature, and excellent consistency has been found. It is shown that the effect of high Tc superconducting film becomes more significant for the values of temperature close to the critical temperature. Also, it is shown that uniaxially anisotropic substrate affects the resonant characteristics of the superconducting microstrip antenna and consequently they must be taken into account in the design stage. Our proposed model can be extended to a circular microstrip patch in an anisotropic ferrite multilayer environment. This model, however, gives a quite good approximation of the superconducting antenna's behavior and leads to a very short calculation time for the analysis.

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