

## Multiparameter Optimization for Electromagnetic Inversion Problem

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### Abstract

Electromagnetic (EM) methods have been extensively used in geophysical investigations such as mineral and hydrocarbon exploration as well as in geological mapping and structural studies. In this paper, we developed an inversion methodology for Electromagnetic data to determine physical parameters of a set of horizontal layers. We conducted Forward model using transmission line method. In the inversion part, we solved multi parameter optimization problem where, the parameters are conductivity, dielectric constant, and permeability of each layer. The optimization problem was solved by simulated annealing approach. The inversion methodology was tested using a set of models representing common geological formations.

### 1. Introduction

Electromagnetic (EM) geophysical methods are used to gain information of the earth's subsurface structure. They are based on the phenomenon of interaction of time varying electromagnetic source fields with the physical properties of the earth. These methods are applicable where there is a sufficient contrast between the electromagnetic properties of the various subsurface units of interest. In these scenarios the electromagnetic properties can act as a surrogate parameter for surface mapping. The most electromagnetic properties that can be resolved by EM methods are Electrical Conductivity, Magnetic Permeability, and Dielectric Permittivity.

In this paper, we present an inversion methodology for EM data of an earth's horizontal layered model with different conductivity, permeability, and permittivity. First, a forward model was constructed based on the Transmission line equivalent method. Then we used simulated annealing as a global optimization tool for the inversion process. We tested the inversion methodology against a set of horizontal layers that represent common geological structures, and the resultant estimated parameters have an allowed error margin, compared with the acceptable range of the model rocks, and sediments.

### 2. The Forward Model

The most widely used earth model when EM methods are considered is the horizontal layers model. The model

represents well some typical geological formations. In this paper we dealt with a two layers model as shown below:

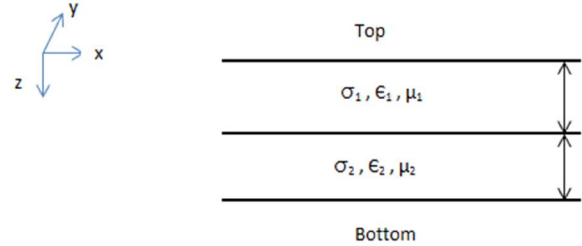


Figure 1: A two horizontal layer model, each layer has depth  $l$ , conductivity  $\sigma$ , dielectric constant  $\epsilon$ , and magnetic permeability  $\mu$ . The top and bottom layers (Boundaries) extent to infinity along  $z$ .

This model can be represented as an equivalent transmission line [1], with two sections corresponds to two layers. As compared to free space, subsurface layers can be considered as impedance lines connected end to end, as illustrated in Figure 2.

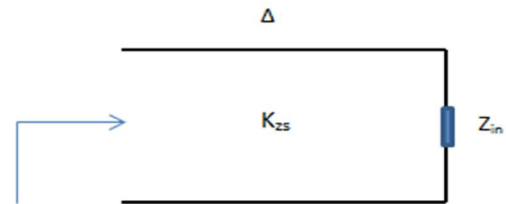


Figure 2: Transmission line equivalent representation of a horizontal layer.

Here, the impedance of the respective layer can be written as:

$$Z_{in} = Z_0 \left[ \frac{Z_L - iZ_0 \tan K_{zs}\Delta}{Z_0 - iZ_L \tan K_{zs}\Delta} \right] \quad (1)$$

where  $\Delta$  is the layer thickness,

$$\text{and } Z_0 = \frac{w\mu_s}{K_{zs}}$$

$$K_{zs} = \sqrt{K^2 - K_x^2}$$

$$K^2 = w^2\mu_s\epsilon_s + iw\mu_s\sigma$$

where,  $Z_0$  is the Characteristic impedance of the equivalent transmission line.

$w = 2\pi f$ , and  $f$  is the operating frequency

$\mu_s$  is the permeability of the layer

$K$  is the wave number

$K_x$  is the wave number in the  $x$  direction

$\epsilon_s$  is the dielectric constant or permittivity of the layer

$\sigma$  is the conductivity of the layer

In the forward model, an unphased infinite wire in the  $y$  direction acts as a transmitter leading to transverse electric field in the  $Z$  direction. Both transmitter and receiver are located at the same point just above the top of the first layer. The thickness of the first layer is 500 meters, and the same for the second layer, giving a total model depth of 1Km. In this paper, we selected three models to represent possible geological configurations. Table 1 summarises the possible permittivity, conductivity, and permeability ranges of these geological formation [2-3].

Table 1: Physical Properties of the four geological layers used in paper.

Geological Layer	Relative Dielectric Constant range [ $\epsilon_r$ ]	Electrical Conductivity range [ $\sigma$ ]	Relative Magnetic Permeability range [ $\mu_r$ ]
Basalt	12	$1 \times 10^{-5}$ to 0.005	1.0002 to 1.175
Granite	4.8 to 18.9	$1 \times 10^{-5}$ to 0.005	1 to 1.05
Sandstone	4.7 to 12	$1.25 \times 10^{-4}$ to $5 \times 10^{-3}$	1 to 1.02
Limestone	4 to 8	$1 \times 10^{-4}$ to $2 \times 10^{-3}$	1 to 1.003

In Table 2, we identify a “spread” factor that quantifies the wideness of each physical parameter for each layer around the average. The spread factor is defined as:

$$spread (\%) = \frac{2(max-min)}{max+min} \times 100 \quad (2)$$

It can be noticed that except for the relative dielectric constant of Basalt, the spread factor of the relative dielectric constant and conductivity of the four layers are quite large, which put a challenge to the inversion algorithm. We applied the forward model approach on three two layered cases as illustrated in Fig. 3, and their properties are presented in Table 3.

Table 2: Spread Factor (%) of each physical parameter of the four geological layers.

Geological Layer	Relative Dielectric Constant Spread factor	Electrical Conductivity Spread factor	Relative Magnetic Permeability Spread factor
Basalt	0	199.20	16.07
Granite	118.98	199.20	4.87
Sandstone	87.42	190.24	1.98
Limestone	66.66	180.95	0.29

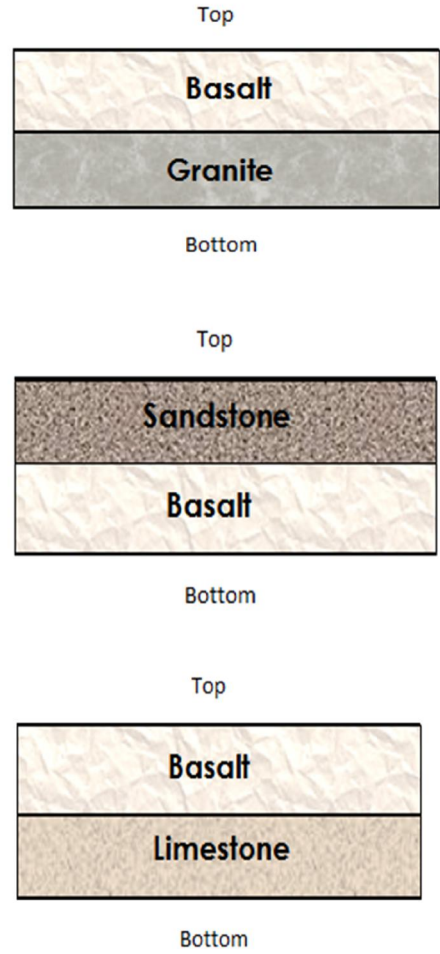


Figure 3: Three horizontal layered models representing Common Geological Formations.

Table 3: Physical Properties of the three Forward case studies used in paper.

Case no.	Geological Formation	Relative Dielectric Constant $\epsilon_r$	Electrical Conductivity $\sigma$	Relative Magnetic Permeability $\mu_r$
1	Basalt	12	0.005	1.05
	Granite	15	0.002	1.03
2	Sandstone	9	$1.25 \times 10^{-4}$	1.015
	Basalt	12	$1 \times 10^{-4}$	1.1
3	Basalt	12	0.002	1.1
	Limestone	5	$1 \times 10^{-3}$	1.002

### 3. The Inversion Algorithm

In the inversion algorithm, we aimed to change the horizontal layer model parameters through an iterative process in order to reduce the misfit between the initial and the actual model. This iterative process was done in the form of global optimization problem. The objective of global optimization [4-7] is to find the globally best solution of nonlinear models, in the presence of multiple local optima.

Simulated annealing method was extensively used in global optimization problem. It mimics the process of annealing to the solution of an optimization problem. When applied, the multiobjective function of the problem is iteratively minimized with the help of the introduction of fictitious temperature, which is, in the optimization cases, a simple controllable parameter of the algorithm [8-9].

Simulated annealing method has the advantage of being adaptable with respect to the evolutions of the problem, and gave good results for a number of problems, especially with many parameters. Multiparameter optimization problem can be formulated as follows: Find the vectors  $\bar{x}^* = [x_1^*, x_2^*, \dots, x_r^*]^T$  of decision variables that simultaneously optimize the S objective values  $\{f_1(\bar{x}), f_2(\bar{x}), \dots, f_S(\bar{x})\}$ , while satisfying the constraints if any. An objective function,  $\mathcal{E}$ , is constructed in the form:

$$\mathcal{E} = \sum_{m=1}^M |E_m^m - E_c^m|^2 \quad (3)$$

Where  $E_m^m$  is the measured electrical field collected at the  $m^{\text{th}}$  wavenumber and  $E_c^m$  is the electric field computed from a guess of what the 3N material properties might be. Hence, the objective function is 3N-dimensional, that is :

$$\mathcal{E} = (\{\epsilon_{rn}\}, \{\sigma_n\}, \{\mu_{rn}\}, n = 1:N) \quad (4)$$

Where  $\epsilon_{rn}, \sigma_n, \mu_{rn}$  are the relative dielectric constant, conductivity, and relative permittivity, respectively, of the  $n^{\text{th}}$  layer. Such a 3N-dimensional function will have many minima. One then looks for the 3N variables that lead to the global minimum of the objective function, which when found will identify the sought after material properties of the two layered model .

To find the layer's physical properties, we convert the inverse scattering problem into a global minimization one. The Electromagnetic fields scattered from the Earth's layers are collected at M wave numbers. The problem formulated to find the six physical properties of the two layer model, given the measured scattered electromagnetic fields at M wavenumbers.

The proposed simulated annealing method is applied to the three studied cases. We started with an initial solution of all zeros, and run the algorithm until stopping minimization criteria is achieved. If the minimization criteria were not satisfied, the algorithm stops after 10,000 iterations, and take the set with the minimum least square error as the answer of the problem. The annealing temperature is decreased as the algorithm proceeds, and the cooling schedule is given by the following formula:

$$T_c = a^c T_0 \quad (5)$$

Where  $c = 1, 2, 3, \dots$  etc. is the iteration number.

$T_0$  is the initial temperature, and  $a$  is the cooling rate "i.e. 0.95".

The results of the algorithm for the three cases are illustrated in Table 4. Also, in this paper we define the term relative percentage error  $\delta$  as follows:

$$\delta = \frac{\text{True value} - \text{Estimated value}}{\text{True value}} \times 100 \quad (6)$$

The error margin for each physical parameter of each layer is presented in Table 4. It can be noticed that the error margin resulted from the inversion methodology is quiet low for most of the estimated parameters. Even in cases when  $\delta$  is relatively high, the large % are far less than the spread factor of the physical parameter. Hence, the a posteriori knowledge is better than the a priori one. Furthermore, the estimated parameter still can be interpreted correctly, as it is within the natural range of it for each of the investigated layers. So the results are quite acceptable from the geological point of view given the non-uniqueness nature of such problems.

Table 4: Estimated Physical Properties of each layer of the three case studies after the inversion algorithm.

Case no.	Relative Dielectric Constant $\epsilon_r$	Electrical Conductivity $\sigma$	Relative Magnetic Permeability $\mu_r$	Minimum Objective Function Value
1	12	0.0048	1.0499	3.1994 x $10^{-7}$
	15.4584	0.0017	1.0031	
2	10.5609	0.0001	1.0133	1.0495 x $10^{-7}$
	12	0.0013	1.0750	
3	12	0.0019	1.1000	3.4871 x $10^{-8}$
	5.0669	0.0017	1.0013	

Table 5: Relative percentage error  $\delta$  (%) of estimated Physical Properties of each layer of the three case studies after the inversion algorithm.

Case no.	Dielectric Constant $\epsilon_r$	Electrical Conductivity $\sigma$	Magnetic Permeability $\mu_r$
1	0.0 %	4%	0.0%
	3.056 %	15%	2.6 %
2	17.34%	0.0%	0.167 %
	0.0%	0.0%	2.27 %
3	0.0%	5%	0.0 %
	1.338%	0.0%	0.0698%

#### 4. Conclusion

In this paper, we implemented an inversion methodology using simulated annealing as a multiparameter optimization technique to get an appropriate solution. The methodology was tested against three cases of horizontal layered model of the Earth. The results are quite promising in estimating the conductivity, permeability, and dielectric constant with an acceptable error margin.

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