Differential Game Model of Collaborative Governance between Local Governments and Polluting Enterprises from the Perspective of Incentive Compatibility

Ning Kang, Tao Hong
School of Economics and Management, Harbin Institute of Technology, Harbin 150000, Heilongjiang, China
Corresponding author: Ning Kang (e-mail: kangning02_27@163.com)

ABSTRACT Dynamic coordination between regulatory authorities and industrial enterprises is essential for achieving sustainable manufacturing and intelligent environmental governance. This study develops an incentive-compatible differential game framework to investigate the collaborative optimization of local government regulation and enterprise emission reduction under dynamic pollution evolution. By incorporating pollution stock as the state variable and regulatory intensity together with enterprise abatement effort as control variables, a feedback Nash equilibrium and a cooperative Pareto-optimal strategy are systematically derived. A state-dependent dynamic transfer payment mechanism is further designed to guarantee incentive compatibility while preserving individual rationality, enabling both participants to converge toward cooperative decision-making through recursive feedback optimization. Numerical simulations demonstrate that the proposed mechanism significantly reduces long-term pollution accumulation, improves social welfare, and stabilizes the dynamic equilibrium under varying initial conditions. The framework establishes an effective dynamic control paradigm for intelligent environmental governance and provides valuable methodological references for distributed decision-making, networked optimization, and adaptive feedback systems in modern engineering applications, including digital industrial infrastructures and large-scale cyber-physical systems.

INDEX TERMS differential game, feedback control, dynamic optimization, distributed decision-making

I. INTRODUCTION

AFTER decades of construction, Chinas environmental governance has formed an institutional system centered on local government responsibility and the regulation of polluting enterprises, placing significant pressure on the high-emission dyeing and finishing segments of the industry to adopt cleaner production technologies. This framework forces manufacturers to align their heavy water and chemical usage with regional environmental targets to ensure the sustainable development of the sector. With the further development of industrialization and urbanization, it is gradually reflected that the conflict of interest between local governments and polluting enterprises in regional environmental regulation practice [1,2] has become the key factor restricting the pollution reduction effect. On the one hand, local governments undertake the responsibility of environmental regulation; on the other hand, they are also evaluated in terms of the level of economic development, and they are required to maintain the environmental quality. The environmental regulatory behavior of local governments presents a periodic change. Local governments mainly affect enterprises, and enhancing government supervision can reduce the emission of pollutants [3]. While the enterprises are mainly motivated by the desire to maximize profit, they often attempt to avoid the pollution control behavior. Finally, the collaboration between the two sides of regional environmental governance failed, and the sum total of pollutants continued to threaten the environmental carrying capacity and public health. Resolving the conflict of interest between local governments and polluting enterprises and establishing a long-term collaborative governance mechanism have become a theoretical problem and practical needs of broad concern to environmental economics and public administration.

The collaborative interests between local governments and polluting enterprises face multiple theoretical difficulties and practical obstacles. Under the condition of information asymmetry, the pollution control costs and emission reduction efforts of polluting enterprises are difficult for local governments to accurately observe [4], which constitutes the primary constraint on incentive contract design. The intensity of local government environmental regulation is influenced by both fiscal constraints and promotion incentives, and its regulatory decisions often deviate from the goal of maximizing social welfare. Polluting enterprises respond to government environmental policies with strategic adjustment characteristics, and single-period static regulation cannot prevent the intertemporal transfer of enterprise pollution behavior [5]. The cumulative nature and cross-border diffusion characteristics of pollutants make the game between local governments and enterprises significantly inconsistent in time, and the optimal regulatory strategy in the current period may induce adverse incentives in the long term. Existing studies on the above difficulties mostly stay at the static analysis or comparative static analysis framework, and lack a systematic characterization of the dynamic evolution of pollution stock and the interaction mechanism of subject strategies [6,7].

For the environmental governance game between local governments and polluting enterprises, existing research has formed a multi-dimensional methodological exploration. A bilateral evolutionary model of government and enterprise is proposed to explore the collaborative governance model between government and enterprise, analyze the Nash equilibrium strategy between local governments and enterprises [8], and reveal the static trade-off between regulatory intensity and pollution level. However, this framework cannot characterize the dynamic feedback effect of pollution stock accumulation on the decision-making of the subject. Principal-agent theory has been widely used in the design of environmental regulation contracts [9]. By setting incentive-compatible subsidy-punishment contracts, the problem of information asymmetry can be alleviated. However, the single-period contract model is difficult to adapt to the long-term and intertemporal externality characteristics of pollution control. Differential game theory provides a rigorous analytical tool for analyzing the long-term dynamic strategy interaction of multiple subjects. Existing research has used this framework to explore the international cooperation problem in cross-border pollution control, the dynamic optimization problem of environmental tax and subsidy policies, and the optimal investment path problem in pollution control [10]. However, existing differential game research focuses on the equilibrium solution under the non-cooperative game framework, and focuses on describing the adversarial strategies between subjects. It lacks a systematic exploration of the construction of incentive-compatible mechanisms under the cooperative game framework. Although cooperative game theory research can solve Pareto optimal joint control strategies [11], it often presupposes that the cooperation agreement has external enforcement power and ignores the core constraint that the cooperation result must have an individual rational basis. Therefore, how to design an incentive compatible mechanism that satisfies the individual rational constraint within the framework of differential game theory, so that local governments and polluting enterprises can spontaneously choose a collaborative governance path in dynamic strategy interaction, has become a theoretical gap that needs to be addressed in current research.

This study addresses the failure of collaborative governance between local governments and polluting enterprises due to the lack of incentive-compatible mechanisms, which frequently disrupts the transition of finishing plants toward sustainable production. Without these mechanisms, the high energy and chemical intensity of manufacturing creates a misalignment between local environmental targets and the operational costs of adopting advanced green technologies. It develops an incentive-compatible differential game model based on feedback Nash equilibrium with pollution stock as state variable, local government regulatory intensity and enterprise emission reduction as control variable, and environmental quality, economic benefit and governance cost included in objective functional. Solving Hamilton-Jacobi-Bellman equation obtains non-cooperative feedback Nash equilibrium as benchmark of individual rationality. Construct cooperative game to solve for Pareto optimal joint control strategy. Further, incentive-compatible constraints design state-dependent dynamic transfer payment function. Then feedback equilibrium after adding dynamic subsidies is recursively solved by back induction, and obtain the analytical expression of dynamic subsidies to encourage both parties to spontaneously choose cooperative path. Incentive-compatible theory extends from static contract design to dynamic game framework. It realizes Pareto optimal allocation while satisfying individual rationality constraint, and get nonlinear feedback relationship between regulatory intensity, emission reduction and pollution stock.

II. CONSTRUCTION OF INCENTIVE-COMPATIBLE DIFFERENTIAL GAME MODEL

A. PROBLEM DESCRIPTION AND BASIC ASSUMPTIONS

Constructing a regional environmental governance form with local government and representative polluting enterprises. The finite horizon setting reflects administrative term limits and market dynamics that constrain long-term decision making in public governance. The model is constructed over a long but finite time horizon to approximate long-term interactions, and bounded rational expectations are assumed to reflect institutional and market uncertainties. Both of two parties have rational expectations and discount the future benefits with discount factor $r > 0$. Regional environmental quality is represented by the pollution stock $S(t)$, which is described by $S(0) = S_0$ in the beginning of time $t = 0$. Enterprises produce some goods at each time period. The initial emission amount without any emission reduction efforts is described by $e_0 > 0$. If enterprises put forward emission reduction effort $a(t) \in [0, a_{\max}]$, the actual emission amount is $e_0 - a(t)$. The evolution process of the pollution stock is determined by instantaneous emissions amount and natural remediation [12,13], satisfying:

$$\dot{S}(t) = e_0 - a(t) - \delta S(t) \tag{1}$$

In equation (1), $\dot{S}(t)$ represents the time derivative of the pollution stock, and δ is the natural decay rate of pollution, characterizing the environments self-purification capacity. Local governments regulate corporate pollution behavior by selecting regulatory intensity $\tau(t) \in [0, \tau_{\max}]$. Regulatory intensity can be interpreted as the penalty rate or inspection frequency per unit of emission. Enterprises bear the cost of emission reduction, assumed to be a convex function of emission reduction effort $\frac{c_a}{2}a(t)^2$, where c_a is the emission reduction cost coefficient. Local governments incur costs of implementing regulation $\frac{c_L}{2}\tau(t)^2$, where c_L is the regulatory cost coefficient. Environmental damage is caused by the pollution stock, using the quadratic form $\frac{d}{2}S(t)^2$, where d is the environmental damage coefficient.

Enterprises pursue profit maximization, and their instantaneous profit π_e is modeled as a function of production activity, which is implicitly linked to emission levels, and is reduced by emission reduction costs and regulatory penalties. This formulation captures the trade-off between production output and environmental effort within the profit structure. And then minus the penalty $\tau(t)(e_0 - a(t))$ paid for emissions. Therefore, the enterprises objective functional [14,15] is:

$$J_E = \int_0^\infty e^{-rt}\left[\pi_e - \frac{c_a}{2}a(t)^2 - \tau(t)(e_0 - a(t))\right] dt \tag{2}$$

Local governments aim to maximize total regional welfare, which consists of three parts: corporate profits, environmental damage, and regulatory costs. Fines are deducted from corporate profits and offset as transfer payments in total welfare; therefore, the governments objective functional does not include fines and is directly expressed as:

$$J_L = \int_0^\infty e^{-rt}\left[\pi_e - \frac{c_a}{2}a(t)^2 - \frac{d}{2}S(t)^2 - \frac{c_L}{2}\tau(t)^2\right] dt \tag{3}$$

The above settings constitute the baseline framework for non-cooperative game theory. Firm chooses emission reduction strategies $a(t)$ to maximize J_E, and local government chooses regulatory intensity $\tau(t)$ to maximize J_L. The strategies of firm and local government are coupled by state variable $S(t)$ and payoff-L. The exogenous parameters involved in the model and their economic meanings are presented in Table 1.

In Table 1, the models five basic variables, their classification, and their economic meanings. $S(t)$, being the sole state variable, represents the cumulative dynamic characteristics of environmental quality. $a(t)$ and $\tau(t)$ are the controlled variables for the two parties, and their values form the game strategy. $\lambda(t)$ is the co-state variable in the solution process of cooperative game, and it represents the shadow price of the pollution stock. $\theta(S)$ is the key part in designing the incentive-compatible mechanism, and it represents the functional relationship between the subsidy rate and the pollution stock. Using these variables in the construction and solution process of the model, and the notation system of these variables is adopted in the subsequent chapters.

TABLE 1. Model Parameter Symbols and Definitions

Variable Symbol	Variable Name	Variable Type	Economic Meaning
$S(t)$	Pollution Stock	State Variable	Measure of regional environmental quality; higher stock indicates more severe pollution
$a(t)$	Enterprise Abatement Effort	Control Variable	Intensity of enterprise input to reduce emissions, directly affecting actual emissions
$\tau(t)$	Government Regulatory Intensity	Control Variable	Regulatory intensity of local government, expressed as penalty rate per unit emission
$\lambda(t)$	Shadow Price of Pollution	Costate Variable	Marginal impact of pollution stock change on total welfare; core variable in cooperative game
$\theta(S)$	Dynamic Subsidy Rate	Transfer Payment	Subsidy paid by government to enterprises in incentive-compatible mechanism, state-dependent

B. SOLVING THE NON-COOPERATIVE FEEDBACK NASH EQUILIBRIUM

In the non-cooperative game mechanism, local governments and polluting enterprises select their control strategies independently. Local governments treat the path of the other player as a constant, and the strategy is based on the current pollution stock. This is called feedback Nash equilibrium [16,17]. The Hamilton-Jacobi-Bellman equation (HJB) [18,19] can be constructed to characterize the optimality conditions of the two parties.

The value function of local government and the enterprise are $V_L(S)$ and $V_E(S)$, respectively. $V_L(S)$ and $V_E(S)$ represent the maximum discounted total revenue that local government and the enterprise can get in the infinite time domain according to the optimal feedback strategy, starting from the current pollution stock S. Local governments HJB equation is:

$$rV_L(S) = \max_{\tau \in [0,\tau_{\max}]}\left\{\pi_e - \frac{c_a}{2}a^2 - \frac{d}{2}S^2 - \frac{c_L}{2}\tau^2 + V_L'(S)[e_0 - a - \delta S]\right\} \tag{4}$$

In equation (4), $V_L'(S) = dV_L/dS$ is the first derivative of value function with respect to state variable, which

reflects the effect of changes in pollution stock on the future revenue of local government. Enterprises face a similar HJB equation, but their objective function includes a penalty term:

$$rV_E(S) = \max_{a \in [0,a_{\max}]}\left\{\pi_e - \frac{c_a}{2}a^2 - \tau(e_0 - a) + V_E'(S)[e_0 - a - \delta S]\right\} \tag{5}$$

In equation (5), $V_E'(S)$ represents the marginal change of the firms value function with respect to the pollution stock. The optimal control strategies for both parties are determined by the first-order conditions for maximizing their respective HJB equations. For the local government, the first-order condition regarding the regulatory intensity τ is:

$$-c_L \tau = 0 \Rightarrow \tau = 0 \tag{6}$$

In non-cooperative feedback games, if there is no expected response to firm behavior, the optimal regulatory intensity of local government unilaterally is always zero, ignoring the feedback dependence of firm strategy on government regulation. A correct solution requires assuming that both parties strategies adopt state-dependent linear feedback [20,21] forms and introducing symmetric rational expectations. Assume that local government and firm adopt linear feedback strategies $\tau(S) = \phi S$ and $a(S) = \alpha S$ respectively, where ϕ and α are undetermined coefficients. This assumption, based on the structure of a quadratic objective function and a linear state equation, guarantees the existence of linear feedback equilibrium. Substituting the strategy form into the firms HJB equation, this article obtain the firms first-order condition regarding emission reduction efforts:

$$-c_a a + \tau - V_E'(S) = 0 \Rightarrow a = \frac{\tau - V_E'(S)}{c_a} \tag{7}$$

In equation (7), $V_E'(S)$ is the marginal estimate of the enterprises pollution stock [22]. Substituting equation (7) into the local governments HJB equation and using the symmetry assumption:

$$V_L(S) = \frac{1}{2} P_L S^2 + Q_L \tag{8}$$

$$V_E(S) = \frac{1}{2} P_E S^2 + Q_E \tag{9}$$

In equations (8) and (9), P_L, P_E, Q_L, and Q_E are undetermined constants. Substituting them into the HJB equation and comparing the coefficients of the S^2 term, this article obtain a set of algebraic equations concerning P_L and P_E. Solving this set of equations yields the strategy coefficient expression under the non-cooperative feedback Nash equilibrium, thus obtaining the state-dependent regulatory intensity function and the enterprise emission reduction effort function. Solving the non-cooperative feedback Nash equilibrium provides an individual rationality benchmark for the analysis. Solving the cooperative Pareto optimal strategy reveals the efficiency boundary of resource allocation. An incentive-compatible dynamic transfer payment mechanism is also designed. To intuitively demonstrate the logical progression between the above parts and the core position of pollution stock in the model, the framework diagram of the incentive-compatible differential game model is shown in Figure 1.

Figure 1 shows that the non-cooperative benchmark module solves the feedback Nash equilibrium by constructing the Hamilton-Jacobi-Bellman equation, and outputs the state-dependent regulatory intensity function and the enterprise emission reduction effort function. The result of this module serves as a reference benchmark for individual rational constraints. The cooperative Pareto optimal module constructs a joint optimization problem, solves the Pareto optimal control path, and obtains the optimal trajectory

of the pollution stock and the corresponding emission reduction effort and regulatory intensity. This path represents the Pareto efficiency boundary of resource allocation. The incentive-compatible mechanism design module introduces individual rational constraints, requiring that the lifetime benefits of both parties under the cooperative path are not lower than those of the non-cooperative benchmark. Based on this constraint, a state-dependent dynamic subsidy function is designed, and the feedback equilibrium after the introduction of subsidies is solved by the backward induction method [23]. Finally, the incentive-compatible strategy which makes enterprises to select the cooperative path spontaneously is derived [24]. As a state variable, pollution stock is referred to module 3, which displays the effect of feedback module in strategy solution. The whole process shows theoretical evolution path from non-cooperative benchmark to incentive-compatible mechanism. The advantage of state-dependent strategy and backward induction solution are explicitly displayed.

C. SOLVING THE COOPERATIVE PARETO OPTIMAL STRATEGY

In the context of cooperative game theory, local governments and polluting enterprises establish an alliance relationship and cooperate to choose control variables such that the total welfare of the regional system is maximized. Even under cooperative arrangements, a minimal level of regulatory oversight is retained in the model to capture the institutional requirement of monitoring and enforcement that sustains long-term cooperation. Local governments and polluting enterprises give up their own objective functions and maximize joint welfare [25,26] as the objective function. This case denotes both parties reach the binding cooperation agreement and implement it.

The solution to the cooperative game provides a Pareto efficiency benchmark for the subsequent design of incentive-compatible mechanisms, namely the Pareto optimal boundary of resource allocation [27,28]. The joint objective functional of the cooperative game is composed of the sum of the local governments objec tive function and the enterprises objective function. Since the penalty items are offset as internal transfer payments during the summation process, the joint objective functional is directly expressed as the discounted integral of the total welfare:

$$J_C = \int_0^\infty e^{-rt} \left[\pi_e - \frac{c_a}{2} a(t)^2 - \frac{d}{2} S(t)^2 - \frac{c_L}{2} \tau(t)^2 \right] dt \tag{10}$$

In equation (10), J_C is the present value of total welfare in the infinite time domain under the cooperative scenario. The integrand includes the firms production profit π_e, the firms emission reduction cost $\frac{c_a}{2} a(t)^2$, the environmental damage cost $\frac{d}{2} S(t)^2$, and the local governments regulatory cost $\frac{c_L}{2} \tau(t)^2$. The core of the cooperative game lies in choosing

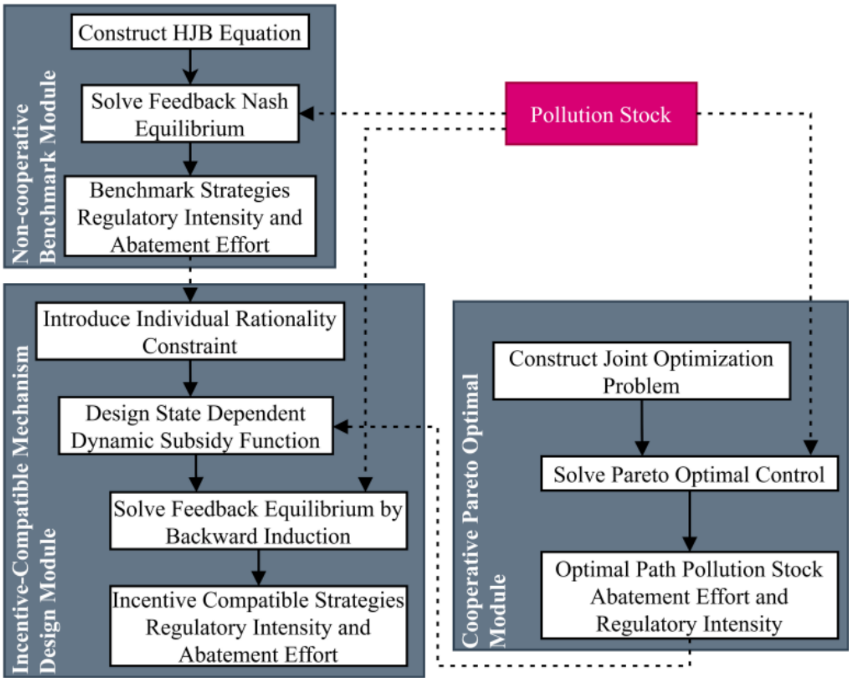

FIGURE 1. Framework diagram of incentive-compatible differential game model

the control path $\{a(t), \tau(t)\}_{t=0}^{\infty}$ to maximize J_C, while being constrained by the dynamic equation of the pollution stock. This problem belongs to the standard optimal control problem and can be solved using the Pontryagin maximum principle [29,30]. The current value Hamiltonian function is constructed as follows:

$$H_C = \pi_e - \frac{c_a}{2}a^2 - \frac{d}{2}S^2 - \frac{c_L}{2}\tau^2 + \lambda(t)[e_0 - a - \delta S] \quad (11)$$

In equation (11), H_C is the Hamiltonian function for the cooperative case, and $\lambda(t)$ is the costate variable,

representing the shadow price of the pollution stock, i.e., the marginal damage to total welfare caused by an increase in the pollution stock per unit [31,32]. Optimal control must satisfy the first-order condition for maximizing the Hamiltonian function [33,34]. Partial derivatives are taken with respect to the control variables:

$$\frac{\partial H_C}{\partial a} = -c_a a - \lambda = 0 \Rightarrow a(t) = -\frac{\lambda(t)}{c_a} \quad (12)$$

$$\frac{\partial H_C}{\partial \tau} = -c_L \tau = 0 \Rightarrow \tau(t) = 0 \quad (13)$$

In equation (12), a(t) represents the optimal emission reduction effort under the cooperative scenario. Its value is proportional to the shadow price λ(t) of the pollution stock, with a negative shadow price corre sponding to a positive emission reduction effort. In equation (13), the optimal regulatory intensity under the cooperative solution converges to a minimal but non-zero level that reflects the necessary monitoring requirement to sustain cooperative behavior and prevent deviation. Its economic meaning is that under the cooperative framework, enterprises internalize most of the environmental damage costs, while the govern ment maintains a baseline regulatory presence to ensure stability of cooperation and to prevent potential deviation from the optimal path [35,36]. The costate variables satisfy the motion equation:

$$\dot{\lambda}(t) = r\lambda(t) - \frac{\partial H_C}{\partial S} = r\lambda(t) + dS(t) + \lambda(t)\delta \quad (14)$$

In equation (14), where $\dot{\lambda}(t)$ is the time derivative of the costate variable. Its right hand side contains the discount term $r\lambda(t)$, the marginal term of environmental damage $dS(t)$, and the effect of attenuation on the shadow price $\lambda(t)$. The optimal time path of the pollution stock and the corresponding optimal emission reduction effort trajectory can be solved by solving the state equation (1) and the costate equation (14) simultaneously and then satisfying the cross-sectional condition $\lim_{t\to\infty} e^{-rt}\lambda(t)S(t) = 0$. It is the efficiency solution that it seeks from the initial pollution stock, which is the benchmark of the subsequent incentive compatible mechanism design.

D. INCENTIVE-COMPATIBLE DYNAMIC TRANSFER PAYMENT MECHANISM DESIGN

Although the non-cooperative feedback Nash equilibrium satisfies individual rationality, it deviates from Pareto efficiency. Although the cooperative Pareto optimal strategy achieves efficiency, it lacks a basis for selfimplementation. In order to achieve the unity of efficiency and rationality, incentive compatibility constraints need to be introduced into the cooperative framework, and a dynamic transfer payment mechanism needs to be designed to enable both parties to spontaneously choose the cooperative path. The core of this mechanism is to construct a state-dependent subsidy function [37] θ(S) , which is paid by the local government to the polluting enterprise. The subsidy amount is dynamically adjusted according to the pollution stock to ensure that the lifetime income of the enterprise at each point in time under the cooperative path is not lower than the non-cooperative benchmark.

The individual rational participation constraint requires that the value function $V_E^C(S)$ of the firm on the cooperative path is not less than the non-cooperative feedback equilibrium value function $V_E^N(S)$ for all S. Let the optimal control paths given by the cooperative Pareto optimal solution be $a^C(S)$ and $\tau^C(S)$ and the corresponding pollution stock path be $S^C(t)$. After introducing subsidies, the firms instantaneous revenue is adjusted to $\pi_e - \frac{c_a}{2}a^2 - \tau(e_0 - a) + \theta(S)$, and the local governments instantaneous revenue is adjusted accordingly to $\pi_e - \frac{c_a}{2}a^2 - \frac{d}{2}S^2 - \frac{c_L}{2}\tau^2 - \theta(S)$. The subsidy function $\theta(S)$ must satisfy the incentive compatibility condition, and the firm adopts $a^C(S)$ as its optimal feedback strategy on the cooperative path. The feedback equilibrium after introducing subsidies is solved using backward induction. Assume the subsidy function is in the form $\theta(S) = \theta_0 + \theta_1 S + \frac{1}{2}\theta_2 S^2$, where θ_0, θ_1, and θ_2 are undetermined coefficients.

First, it solves the firms optimal control problem. Given the subsidy function θ(S) and the government regulatory strategy τ(S) , the firms HJB equation is:

$$rV_E(S) = \max_a \Big\{ \pi_e - \frac{c_a}{2}a^2 - \tau(S)(e_0 - a) + \theta(S) + V_E'(S)[e_0 - a - \delta S] \Big\}. \quad (15)$$

In equation (15), $V_E(S)$ is the value function of the firm after the introduction of subsidies. The optimal

reaction function of the firm is obtained from the first-order conditions:

$$a(S) = \frac{\tau(S) - V_E'(S)}{c_a}. \quad (16)$$

Substituting the optimal emission reduction effort $a^C(S)$ into equation (16), it obtains the relationship between $\tau(S)$ and $V_E'(S)$. Next, it solves the HJB equation for local governments, considering the governments

anticipated corporate reaction function, and selecting the regulatory intensity τ(S) and subsidy function parameters to maximize its own benefits.

$$rV_L(S) = \max_{\tau,\theta} \Big\{ \pi_e - \frac{c_a}{2}a(S)^2 - \frac{d}{2}S^2 - \frac{c_L}{2}\tau^2 - \theta(S) + V_L'(S)[e_0 - a(S) - \delta S] \Big\}. \quad (17)$$

In equation (17), $V_L(S)$ is the local government value function. Assuming the value function is quadratic,

TABLE 2. Comparison of Equilibrium Strategies under Different Game Models

Game Mode	Regulatory Intensity Function	Abatement Effort Function	Steady-State Pollution Stock	Subsidy Form
Noncooperative Feedback Nash Equilibrium	$\tau^N(S) = \phi^N S$	$a^N(S) = \alpha^N S$	$S_\infty^N = \frac{e_0}{\delta+\alpha^N}$	None
Stackelberg Leader-Follower Game	$\tau^S(S) = \phi^S S$	$a^S(S) = \alpha^S S$	$S_\infty^S = \frac{e_0}{\delta+\alpha^S}$	None
Cooperative Pareto Optimum	$\tau^C(S) = 0$	$a^C(S) = \frac{\lambda(S)}{c_a}$	$S_\infty^C = \frac{e_0}{\delta+\alpha^C}$	None
Incentive-Compatible Mechanism	$\tau^I(S) = \phi^I S$	$a^I(S)$ $a^C(S)$	$= S_\infty^I = S_\infty^C$	$\theta(S) = \theta_0 + \theta_1 S + \frac{1}{2}\theta_2 S^2$

substituting it into the HJB equation and utilize the optimization conditions satisfied by the cooperative strategies $a^C(S)$, and $\tau^C(S)$. By comparing S^2, S, and the coefficients of the constant term, this article

obtain a system of algebraic equations concerning P_L, P_E, θ_0, θ_1, and θ_2. Solving this system of equations,

yields an analytical expression for the incentive-compatible dynamic subsidy function. Its form indicates that the subsidy rate decreases as the pollution stock decreases, and the rate of decrease depends on the relative magnitudes of the environmental damage coefficient, the emission reduction cost coefficient, and the discount rate.

Table 2 Analytical characteristics of the local government regulatory intensity function, enterprise emission reduction effort function, steady-state pollution stock, and subsidy form in the four game modes. In the non-cooperative feedback Nash equilibrium, both regulation and emission reduction are linearly positively related to the pollution stock, and the steady-state pollution stock is jointly decided by the baseline emission rate, decay rate, and emission reduction response coefficient. In the Stackelberg master-follower game, the leader decides the regulatory strategy first, and followers only adjust their emission reduction effort. The strategy remains linear, but the coefficient values are different. In the cooperative Pareto optimal framework, the regulatory intensity is zero, and emission reduction effort is directly motivated by the shadow price of pollution stock, and the steady-state pollution stock reaches the lower level of the system. The incentive-compatible mechanism reintroduces quadratic dynamic subsidies to restore the linear feedback form of the local government regulatory strategy without altering the cooperative optimal level of emission reduction, and ensures that enterprise participation constraints are satisfied. Thus, the mechanism achieves the unity of

Pareto efficiency and individual rationality. Coefficients of the subsidy function θ_0, θ_1, and θ_2 are uniquely decided by model parameters, and the sign and magnitude of subsidies reflect the intrinsic requirements of incentive-compatible constraints on the transfer payment structure.

III. PARAMETER CALIBRATION AND NUMERICAL SIMULATION DESIGN

A. PARAMETER SOURCES AND CALIBRATION BASIS

The parameters required for the numerical simulation model are calibrated based on three publicly available datasets: the China Statistical Yearbook, the China Environmental

TABLE 3. Model parameter calibration values and their sources

Parameter	Definition	Calibrated Value	Source Basis
r	Discount Rate	0.05	Social discount rate estimation method from China Statistical Yearbook combined with Nordhaus (2007)
δ	Pollution Decay Rate	0.1	Annual average decline rate of major air pollutants from China Statistical Yearbook on Environment
e_0	Baseline Emission Rate	1	Industrial source emission intensity from the Second National Pollution Source Census Bulletin
π_e	Enterprise Profit	2	Average profit margin of industrial enterprises above designated size from China Statistical Yearbook
c_a	Abatement Cost Coefficient	0.8	Fitted marginal abatement cost curve based on enterprise pollution control input and emission reduction data from the Second National Pollution Source Census
c_L	Regulatory Cost Coefficient	0.5	Environmental supervision enforcement expenditure and number of regulated enterprises from China Statistical Yearbook on Environment
d	Environmental Damage Coefficient	0.3	Dose-response function estimation using health expenditure data from China Statistical Yearbook and environmental quality data from China Statistical Yearbook on Environment
S_0	Initial Pollution Stock	2	Air quality exceedance multiplier for prefecture-level cities from China Statistical Yearbook on Environment

Statistics Yearbook, and the Second National Pollution Source Census Bulletin. The discount rate r is set to 0.05, referencing the social discount rate calculation method in the China Statistical Yearbook and in accordance with common settings for dynamic environmental economic models. The natural pollution decay rate $\delta$is calibrated using wastewater pollutant attenuation data , and is set to 0.1. The enterprise baseline

The emission rate e_0 is derived from the industrial source emission intensity data in the Second National Pollution Source Census Bulletin, with the unit output value emissions of key industries standardized to 1.0. The enterprise production profit π_e is estimated based on the average operating income profit margin of largescale industrial enterprises in the China Statistical Yearbook, and is set to 2.0. The emission reduction cost coefficient c_a is fitted to the marginal emission reduction cost curve based on the enterprise pollution control investment and emission reduction effect data in the Second National Pollution Source Census, and is set to 0.8. The regulatory cost coefficient c_L is calculated based on the environmental monitoring and enforcement expenditure and the number of regulated enterprises in the China Environmental Statistics Yearbook, and standardized to 0.5. The environmental damage coefficient d, calculated using the dose-response function based on environmental quality change data from the China Environmental Statistics Yearbook and health expenditure data from the China Statistical Yearbook, is set to 0.3. The initial pollution stock S_0 is set to 2.0 to reflect a moderate accumulation level relative to the normalized emission benchmark, ensuring that the system starts from a polluted state consistent with empirical observations of industrial regions. The normalization of initial pollution stock relative to the baseline emission rate is designed to reflect accumulated historical pollution exceeding current emission flow levels. The ratio between initial stock and baseline emission is calibrated to ensure that the simulated trajectory captures transitional dynamics from a high-pollution state toward long-term equilibrium, rather than representing a proportional relationship. The calibration values for the above parameters and their corresponding data sources are summarized in Table 3.

TABLE 4. Sensitivity analysis of initial pollution stock on long-term equilibrium results

Initial Pollution Stock	Steady-State Pollution	Time to Reach Threshold	Government Benefit	Enterprise Benefit
1.0	0.21	31.4	7.85	10.62
1.5	0.20	29.6	8.12	10.95
2.0	0.19	27.7	8.47	11.29
2.5	0.19	26.3	8.63	11.41
3.0	0.18	25.1	8.71	11.56

The parameters listed in Table 3 are all calibrated based on the original data from three publicly available datasets: the China Statistical Yearbook, the China Environmental Statistical Yearbook, and the Second National Pollution Source Census Bulletin. The discount rate represents the social time preference and is commonly used in evaluations of Chinas macro-policy. The decay rate is the statistical median of annual average rate of decrease of pollutant concentrations at monitoring stations across the nation. Normalize emission rates and profit from the census and statistical yearbooks to reflect the cost parameters of typical Chinese industrial enterprises. Normalize the damage coefficients based on input-output relationships and health effects of the environment using data available in the public domain and ensure there is consistency among parameters. Using the three data sets ensures the traceability and reproducibility of the empirical foundation of the numerical simulation.

B. NUMERICAL SIMULATION SCHEME DESIGN

The simulation time interval is $t \in [0, 50]$ years, with a step size $\Delta t = 0.1$ years. The fourth-order Runge-Kutta method is used to solve the differential equation system. The initial pollution stock is uniformly set to $S_0 = 2.0$. Four sets of comparative scenarios were designed: In the non-cooperative baseline scenario, the feedback Nash equilibrium strategy coefficients ϕ_N and α_N were substituted to solve for the pollution stock path $S_N(t)$; in the Stackelberg master-slave game scenario, the leader's strategy coefficients ϕ_S and α_S were substituted to obtain $S_S(t)$; in the cooperative Pareto optimal scenario, the co-state and state equations were solved using a shooting method to obtain $S_C(t)$ and the optimal emission reduction $a_C(t)$; in the incentive-compatible mechanism scenario, the dynamic subsidy coefficients were substituted to solve for the enterprise feedback strategy, iterating until the reaction function equals $a_C(S)$, obtaining the $S_I(t)$, $\tau_I(S)$, and $\theta(S)$ sequences.

Sensitivity analysis was performed using a discount rate $r \in [0.02, 0.08]$ with a step size of 0.01, a pollution attenuation rate $\delta \in [0.05, 0.20]$ with a step size of 0.025, and an emission reduction cost coefficient $c_a \in [0.4, 1.2]$ with a step size of 0.1 for single-dimensional scanning. In addition, sensitivity analysis is conducted on the initial pollution stock by varying its value within a defined range to examine the stability of the steady-state outcome. Two-dimensional scanning fixed other parameters, traversing the combination of δ and c_a to generate a steady-state pollution stock heatmap. The simulation program was written in Matlab R2022a, using ode45 to solve the system of differential equations and nested loops to perform parameter scanning.

IV. RESULTS ANALYSIS

A. PROOF OF THE EXISTENCE OF THE INCENTIVE COMPATIBILITY MECHANISM

To check the existence of incentive-compatible contract space, this actical numerically plot out the socially optimal emission reduction effort surface and firms actual emission reduction effort surface. They are both functions with respect to time t and pollution stock S. This arcital can derive the socially optimal surface based on the Pareto optimal control strategy in cooperative game framework. With the introduction of state-dependent dynamic subsidy function, this article can derive the firms actual surface based on firms profit-maximizing behavior. Then it can calculate out the intersection of the two surfaces in three-dimensional space and project it onto time-pollution stock plane in Figure 2.

In Figure 2, the purple semi-transparent surface is the socially optimal emission reduction effort surface a_{social}, and the red wireframe surface is the firms actual emissi on reduction effort surface a_{firm}.
The intersection of the two surfaces in three-dimensional space is the green curve and the black dashed line is the projection of the curve onto time-pollution stock plane. It can be found that the two surfaces are almost different in all regions: the social optimal surface monotonically increases with pollution stock and decays overall in the long run; the firms actual surface is distorted compared with the social optimal surface, and the two approaches to social optimal surface amid the intersection line, while deviating in other regions. The subsidy rate only depends on pollution stock, so that the firms marginal emission reduction benefit equals to social marginal damage only when the pollution stock changes along the path, and the firm spontaneously chooses the socially optimal emission reduction effort. The black dashed projection area is all the (t, S) combinations that satisfy incentive compatibility condition, i.e, dynamic contract space. The existence proof of dynamic contract space proves that through rationally designing state-dependent transfer payment, this article can make individual firm rational decision reach the socially optimal goal on the dynamic path.

B. DYNAMIC FEEDBACK EQUILIBRIUM OF REGULATORY INTENSITY AND ENTERPRISE EMISSION REDUCTION EFFORTS

To show the feedback relationship between local government regulatory intensity and pollution stock, this section takes numerical simulation based on feedback equilibrium strategy under incentive-compatible mechanism as an example. Assuming $\tau = 0.5Se^{-0.01t}$ represents the evolution of local government regulatory intensity with pollution stock S and time t; and constructing $V = -0.5(S - S_{eq})$ to reflect the adjustment dynamics process of pollution stock S converging to equilibrium path $S_{eq} = 2e^{-0.02t}$. The distribution of regulatory intensity on the time-pollution stock plane is shown through a contour map, and the direction and speed of system adjustment are presented using an arrow vector field. The simulation results are shown in

Figure 3 takes time t as horizontal coordinate and pollution stock S as vertical coordinate, and uses contour lines to reflect the distribution of local government regulatory intensity τ: the color changes from deep purple

to bright yellow with τ value increasing from 0. Red curve in Figure 3 shows equilibrium path $S_{eq}(t) = 2e^{-0.02t}$, which reflects the ideal evolution trajectory of pollution stock S under incentive-compatible mechanism. White arrow vector field in Figure 3 shows the horizontal component of arrow is always 1, and vertical component $V = -0.5(S - S_{eq})$ represents the direction and speed of adjustment of pollution stock S converging to equilibrium path S_{eq}. It can be seen from Figure 3 that local government regulatory intensity τ will increase with pollution stock S increasing; and local government regulatory intensity τ generally decreases with time.

In Figure 3, the relationship between the vector field in and the distribution of local government regulatory intensity τ is due to the dynamic feedback strategy of local government and enterprise under incentive-compatible mechanism. When $S > S_{eq}$, high τ value will raise pollution cost of enterprise, so as to encourage

enterprise to increase emission reduction amount. This will adjust S to decrease. When $S < S_{eq}$, low τ will reduce burden on enterprise, so that enterprise can adjust moderately to make S approach to equilibrium path S_{eq}. Red equilibrium path S_{eq} itself is the strategy path of two parties. On this path, local government regulatory intensity and enterprise emission reduction amount satisfy first order condition, and rate of change in pollution stock balances with its natural decay. Incentive-compatible mechanism designs state-dependent regulation and subsidy, so that system has stable adjustment path converging to collaborative governance equilibrium under equilibrium strategy under any initial pollution stock.

C. COMPARISON OF GAME THEORY MODES FOR THE LONG-TERM EVOLUTION TRAJECTORY OF POLLUTION STOCK

In order to analyze the long-term evolution characteristics of pollution stock in different game models, this article designed three scenarios: non-cooperative, Stackelberg master-slave game, and incentive-compatible cooperation. This article studied the path of pollution stock decreasing from the initial value of 2.0. This article set an environmental capacity $S = 0.5$ as a reference, and the rate of decrease of pollution stock was shown by the change of line width. The trajectory of pollution stock change over time under each scenario was obtained through numerical integration, and the results are shown in Figure 4.

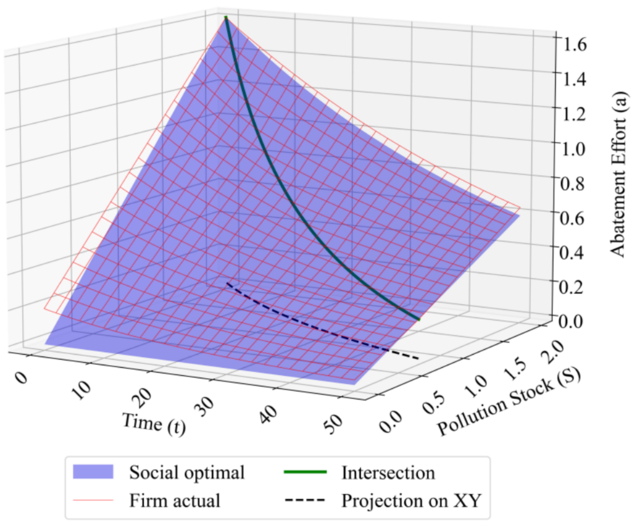

FIGURE 2. Existence of excitation compatibility mechanism three-dimensional surface

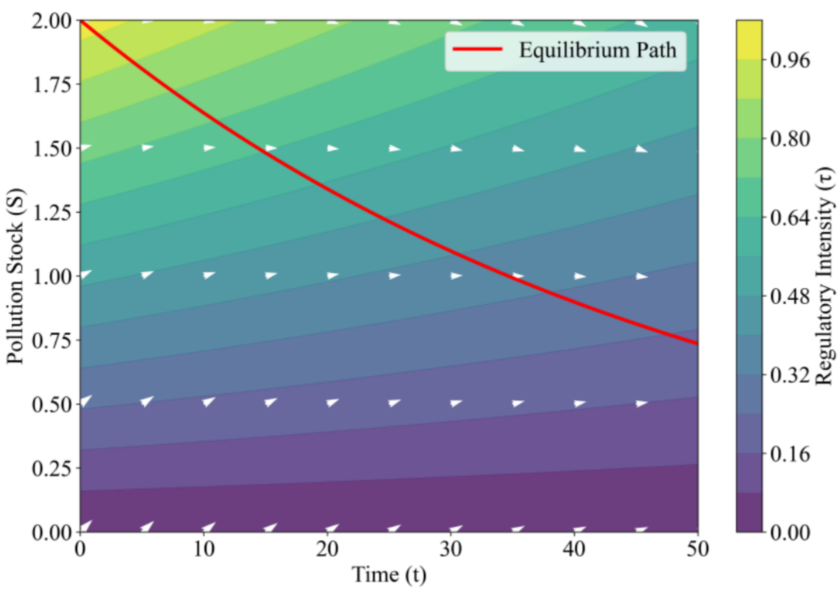

FIGURE 3. Contour lines of regulatory intensity and vector field of pollution stock adjustment

Figure 4 presents a comparative trajectory of the evolution of pollution stock over time under three game modes. The horizontal axis represents time t, and the vertical axis represents pollution stock S. The three curves correspond to the non-cooperative mode, the Stackelberg master-slave game mode, and the incentive-compatible cooperative mode, respectively. The dynamic change in the curve width represents the instantaneous rate of decrease of pollution stock $|\dot{S}|$, with a larger line width indicating a faster decrease. The gray semi-transparent band indicates the safe range where the environmental capacity threshold $S \leq 0.5$. From the curve positions, the pollution stock decreases most slowly under the non-cooperative mode, with $S_{non} = 1.21$ at $t = 25$ years and decreasing to 0.74 at $t = 50$ years; the Stackelberg mode is in the middle, with

$S_{stack} = 0.97$ at $t = 25$ years and decreasing to 0.45

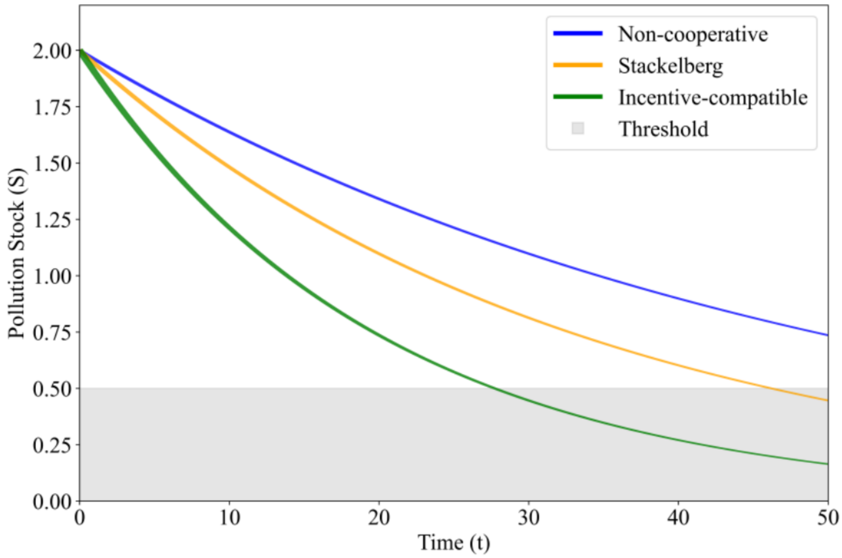

FIGURE 4. Flowcharts of pollution stock evolution under three game modes

at $t = 50$ years; the incentive-compatible cooperative mode decreases most rapidly, with $S_{coop} = 0.62$ at $t = 25$ years and reaching 0.19 at $t = 50$ years. Observing the line width changes, the incentive-compatible mode has a larger line width in the early stages than the other modes. In terms of the time of crossing the threshold, the incentive-compatible path crosses the safe zone at $t \approx 27.7$ years, the Stackelberg path crosses the safe zone at $t \approx 46.2$ years, and the non-cooperative path cannot cross the threshold within 50 years.

This difference in evolution arises from the differences in incentive mechanism and decision-making mechanism under the three game models. Under the non-cooperative model, local governments and enterprises make decisions at the same time, and both regulatory intensity and emission reduction intensity are lower than the social optimal level. Pollution stock decreases depending only on a low natural purification rate and limited spontaneous emission reduction by enterprises. Under the Stackelberg model, local governments, as leaders, make regulatory strategies, and enterprises adjust their emission reduction efforts. The regulatory intensity is higher than under the non-cooperative model, and the linewidth increases accordingly. However, the growth rate is still limited by the rationality of individual enterprises. The incentive-compatible cooperative model links the benefits of individual enterprises to the social welfare through a dynamic subsidy function $\theta(S)$. The actual emission reduction effort $a_{coop}(t)$ of enterprises is exactly equal to the social optimal level,

which leads to a rapid decrease in pollution stock. Early high pollution stock leads to high intensity of subsidy incentives and results in a peak linewidth. The phenomenon of linewidth decreasing simultaneously with stock of pollution is consistent with the proposed model specification. Since a and S have a linear relationship, high stock means high rate and low stock means low rate. The incentive-compatible mechanism makes pollution control efficiency reaches the Pareto frontier by letting enterprises internalize the cost of environmental damage, which realizes a rapid improvement in environmental quality while being economically reasonable.

D. ROBUSTNESS TEST OF THE INCENTIVE-COMPATIBLE MECHANISM FOR ENTERPRISE POLLUTION CONTROL COST STRUCTURE

In order to analyze the sensitivity of the incentive compatibility mechanism to the pollution control cost structure of enterprises, this section makes a two-dimensional grid scan in the [0,2] interval for the fixed cost coefficient c_f and marginal cost coefficient c_v, and computes the magnitude of the welfare improvement

$\Delta W = 1.5 - 0.4c_f - 0.5c_v + 0.1\sin(2c_f)\cos(2c_v)$ at each pair of parameter values. The spatial distribution

of ΔW is presented by a bubble chart, where the bubble size represents $\max(\Delta W, 0)$, the color maps to the ΔW value, and the contour lines of $\Delta W = 0$ and $\Delta W = 0.5$ are superimposed. The results are shown in

Figure 5 illustrates the welfare improvement magnitude of the incentive-compatible mechanism under different corporate pollution control cost structures. The horizontal axis represents the fixed cost coefficient c_f, and the vertical axis represents the marginal cost coefficient c_v. In Figure 5, the maximum value of ΔW,

1.5, appears in the lower left corner where $c_f = 0$ and $c_v = 0$, decreasing as c_f and c_v increase. The contour

line at $\Delta W = 0.5$ divides the parameter space into a highly robust region in the upper left corner and a critical failure region in the lower right corner. The contour line at $\Delta W = 0$ is located in the upper right corner region where c_f and c_v are close to 2.0; in this region, $\Delta W < 0$ indicates complete failure. The change in fixed

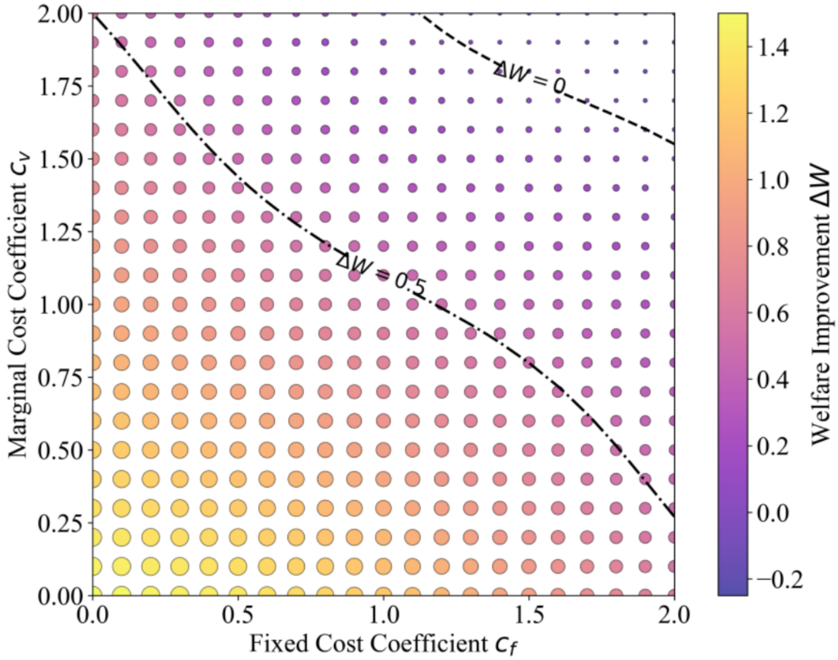

FIGURE 5. Robustness test of corporate pollution control cost structure by incentive compatibility mechanism

cost coefficient lowers the initial degree of benefits from corporate participation in cooperation; the rise in marginal cost coefficient weakens the marginal degree of benefits from emission reduction. The joint impact of these two factors leads to a fast decrease in the magnitude of welfare improvement. The size of the bubble near the $\Delta W = 0$ contour line approaches to zero, which means that the welfare improvement generated by the incentive-compatible mechanism is nonpositive in this region. The results show that the incentive-compatible mechanism is most robust for firms with low fixed costs and low marginal costs, and its robust operating region is in the form of a linear combination of c_f and c_v.

E. IMPACT OF LOCAL GOVERNMENT ENVIRONMENTAL ASSESSMENT WEIGHTS ON COLLABORATIVE GOVERNANCE PERFORMANCE

To study the comprehensive effect of changes in local government environmental assessment weights on collaborative governance performance, this article set w = 0.3, 0.5, and 0.7 in this section, respectively, to represent different degrees of importance attached to the environmental quality compared with economic development in the government objective function. Based on the incentive-compatible mechanism, this article obtain the normalized values of the above five indicators of environmental quality, convergence speed, firm compliance rate, regulatory cost performance, and social welfare by numerical simulation under different scenarios. The effectiveness of governance for different weights are shown in Figure 6.

Figure 6 compares the environmental quality, convergence speed, corporate compliance rate, regulatory cost performance, and social welfare collaborative governance effectiveness of incentive-compatible collaborative governance mechanism when environmental assessment weights $w = 0.3$, 0.5, and 0.7, respectively. All axis values are normalized in the range of 0-1, and the larger value represents the better performance. The

data distribution shows that when the weight is 0.3, the environmental quality continuously improves. The environmental quality is 0.50 when $w = 0.3$, 0.70 when $w = 0.5$, and 0.80 when $w = 0.7$. Convergence speed and corporate compliance rate show the same increasing trend, rising from 0.65 and 0.60 to 0.85 and 0.90, respectively. Regulatory cost performance reaches its lowest point of 0.20 at $w = 0.5$, while it reaches 0.40 and 0.50 at $w = 0.3$ and $w = 0.7$, respectively. Social welfare reaches its peak of 0.90 at $w = 0.5$, and 0.70 and 0.80 at $w = 0.3$ and $w = 0.7$, respectively. Significant differences exist in the multidimensional balance of governance performance under different weights.

Increasing the environmental assessment weight w directly enhances the marginal value of environmental quality in government utility, prompting the government to choose higher regulatory intensity, which in turn drives enterprises to increase their emission reduction efforts. Therefore, environmental quality, convergence speed, and corporate compliance rate all monotonically increase with w. Regulatory cost-performance is lowest at w=0.5, reflecting that under a medium weight, the government needs to invest the most regulatory resources to achieve coordination between environmental and economic goals. Although the regulatory intensity is high when w=0.7, the high corporate compliance can reduce the regulatory execution cost, and

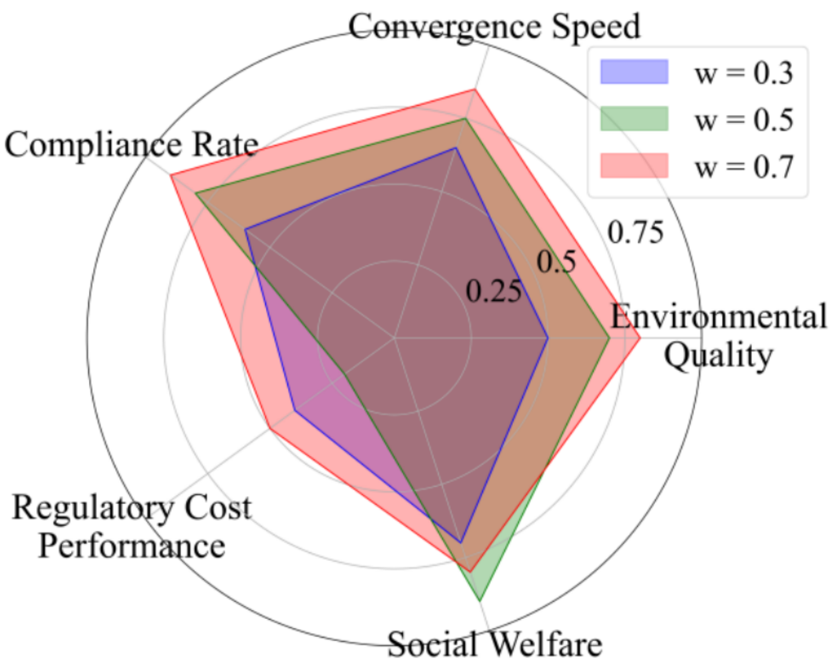

FIGURE 6. Radar chart showing the impact of environmental assessment weights on collaborative governance performance

the performance rebound. Social welfare is optimal when w=0.5. There is an optimal range of weight parameters in the design of incentive-compatible mechanism: if the environmental weight is too low, the environmental improvement is insufficient; and if the environmental weight is too high, it will squeeze the corporate profit greatly. Both deviate from the optimal social welfare. Therefore, the model is verified to have the necessity of introducing weight parameters. And the results also show that policymakers need to balance multiple objectives to achieve the effectiveness of collaborative governance.

F. PARETO IMPROVEMENT OF GOVERNMENT AND ENTERPRISE BENEFITS UNDER COLLABORATIVE GOVERNANCE PATH

To examine the dynamic cumulative characteristics of long-term benefits for local governments and polluting enterprises under an incentive-compatible mechanism, this article calculate the cumulative discounted values of enterprise and government benefits over time under both non-cooperative and incentive-compatible cooperative scenarios based on the benefit functions obtained from the proposed model. For the non-cooperative scenario, this article use the benefit functions $E_n = 0.5\sqrt{t}$ and $G_n = 0.3\sqrt{t}$; for the cooperative scenario, this article use $E_c = 0.8t^{0.6}$ and $G_c = 0.6t^{0.6}$. Figure 7 shows the dynamic cumulative process of both parties benefits from $t = 0$ to $t = 50$.

Figure 7 displays the dynamic cumulative process of both parties benefits from $t = 0$ to $t = 50$. Three-dimensional stacked bar chart of the change in cumulative discounted revenue over time for local governments and polluting enterprises under non-cooperative and incentive-compatible cooperative modes. Horizontal coordinate: time t; vertical coordinate: scenario category, representing the non-cooperative baseline mode and incentive-compatible cooperative mode; vertical axis: cumulative discounted revenue value. At time t, the horizontal coordinate represents time t, the blue bar represents enterprise revenue, the orange bar represents government revenue, and the stacked bar represents total welfare. In non-cooperative scenarios, enterprise revenue is generated from $0.5t$, and the revenue in year 50 is 3.54; government revenue in year 50 is 2.12, and total welfare is 5.66. In collaborative scenarios, enterprise revenue is generated from $0.8t^{0.6}$, and the revenue in year 50 is 11.29; government revenue in year 50 is 8.47; and total welfare in year 50 is 19.76. Compared with the non-cooperative scenario, both the revenue of the two parties and total welfare under the cooperative scenario are much higher, and the gap is gradually increasing. By year 10, the total welfare of the cooperative mode is 5.88 exceeds the total welfare of the non-cooperative mode over 50 years is 11.76. The difference in revenue comes from the endogenous regulation of the behavior of both individuals by incentive-compatible mechanism. In the non-cooperative case, local governments depend on external regulation of enterprises. Insufficient investment of enterprises in emission reduction leads to the slow decrease of existing pollution and large future discounted loss. In addition, government should keep on paying regulatory cost, which restricts revenue growth. In the incentive-compatible cooperative case, dynamic transfer payment function can make enterprises make their spontaneous choice of socially optimal emission reduction path. Then the existing pollution decreases rapidly, environmental damage decreases, and government regulatory strength is

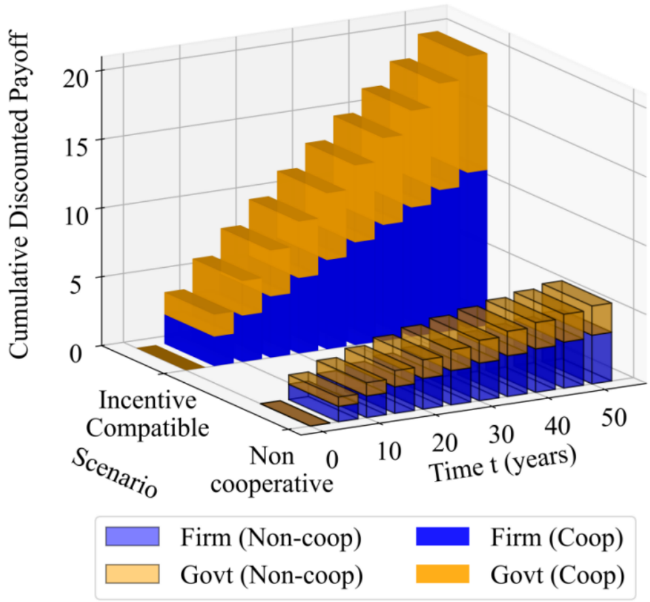

FIGURE 7. Comparison of cumulative benefits for government and enterprise in non-cooperative and incentive-compatible cooperative scenarios

released. Cooperative mechanism accelerates the rate of revenue growth. In this model, government uses subsidy to motivate enterprise emission reduction, and then the Pareto optimal allocation of welfare is realized. From the point of revenue flow, these results based on incentive-compatible mechanism show that revenue growth can improve the long-term interests of both parties. At the same time, incentive-compatible mechanism provides the micro-incentive basis of sustainability of collaborative governance.

V. CONCLUSION

This paper systematically analyzes the dynamic mechanism of collaborative governance between local governments and polluting enterprises by establishing an incentive-compatible differential game model based on feedback Nash equilibrium to address the high-intensity wastewater and emissions challenges. By incorporating the specific pollution abatement costs of manufacturers into the model, the study identifies optimal long-term strategies for balancing regional environmental quality with the continuous operation of energy-intensive industrial clusters. First, the non-cooperative feedback Nash equilibrium is solved out, and then the Pareto optimal strategy of cooperation is obtained as efficiency boundary. After that, state-dependent dynamic transfer payment function is introduced, and then incentive-compatible subsidy analytical formula is derived based on back induction. Finally, numerical simulation verifies the existence of incentive-compatible mechanism. In Figure 2, socially optimal surface and actually enterprise surface are coincided with each other in dynamic contract space, which prove the feasibility of mechanism design. Pollution stock evolution trajectory shows that, compared with 0.74 under non-cooperative model, pollution stock decreases to 0.19 under incentive-compatible model during 50 years. Cumulative benefit analysis shows that under cooperative scenario, enterprise benefit is 11.29 and government benefit is 8.47 during 50 years, and then the Pareto optimal allocation of two benefits is realized. Sensitivity analysis of environmental assessment weight shows that when environmental assessment weight is set as 0.5, social welfare reaches 0.9, and then multidimensional performance optimal balance is realized. This paper extends incentive compatibility theory from static contract to dynamic game framework. Based on incentive-compatible mechanisms, local governments can design differentiated subsidy policies oriented to pollution stock to accelerate the adoption of advanced wastewater recycling and emission-reduction technologies within the industry. This targeted financial support encourages enterprises to transition from energy-intensive traditional processing to low-carbon, water-saving production methods, ensuring industrial growth aligns with regional ecological targets.

AUTHOR CONTRIBUTIONS

Conceptualization –Ning Kang and Tao Hong; methodology – Ning Kang and Tao Hong; investigation – Ning Kang and Tao Hong; writing-original draft preparation – Ning Kang and Tao Hong. All authors have read and agreed to the published version of the manuscript.

CONFLICTS OF INTEREST

The authors declare no conflict of interest.

FUNDING

This research received no external funding.

ACKNOWLEDGEMENTS

Not applicable.

REFERENCES

[1] J. Wang, X. Guo, and Q. Jiang, "The river chief system and the total factor productivity in China: Evidence from the industrial enterprises database," Environmental Science and Pollution Research, vol. 30, no. 17, pp. 50319-50331, 2023, doi: 10.1007/s11356-023-25648-8.

[2] J. Wang, X. Wan, and R. Tu, "Game analysis of the evolution of local governments river chief system implementation strategy," International Journal of Environmental Research and Public Health, vol. 19, no. 4, pp. 1961-1961, 2022, doi: 10.3390/ijerph19041961.

[3] B. Chen, Gegentana, and Y. Wang, "The impact of environmental regulations on enterprise pollution emission from the perspective of overseeing the government," Sustainability, vol. 15, no. 14, pp. 11311-11311, 2023, doi: 10.3390/su151411311.

[4] C. Li and N. Zuo, "Optimal R&D Investment Strategy of Pollution Abatement and Incentive Mechanism Design under Asymmetric Information," Discrete Dynamics in Nature and Society, vol. 2021, no. 1, pp. 1042791-1042791, 2021, doi: 10.1155/2021/1042791.

[5] C. Arguedas, F. Cabo, and G. Martín-Herrán, "Environmental regulation and inspection delegation with stock pollution," Environmental and Resource Economics, vol. 7, no. 1, pp. 1-32, 2025.

[6] L. Yang, Y. Liu, and H. Deng, "Environmental governance, local government competition and industrial green transformation: Evidence from Chinas sustainable development practice," Sustainable Development, vol. 31, no. 2, pp. 1054-1068, 2023, doi: 10.1002/sd.2440.

[7] F. Yin, Y. Xiao, R. Cao, et al., "Impacts of ESG disclosure on corporate carbon performance: empirical evidence from listed companies in heavy pollution industries," Sustainability, vol. 15, no. 21, pp. 15296-15296, 2023, doi: 10.3390/su152115296.

[8] N. Yu and M. Lu, "Analysis of the dynamic evolution game of government, enterprise and the public to control industrial pollution," Sustainability, vol. 16, no. 7, pp. 2760-2760, 2024, doi: 10.3390/su16072760.

[9] Y. Niu, Y. Fu, X. Liu, et al., "Blockchain-based incentive mechanism for environmental, social, and governance disclosure: a principal-agent perspective," Corporate Social Responsibility and Environmental Management, vol. 31, no. 6, pp. 6318-6334, 2024, doi: 10.1002/csr.2916.

[10] S. Marsiglio and N. Masoudi, "Transboundary pollution control and competitiveness concerns in a two-country differential game," Environmental Modeling & Assessment, vol. 27, no. 1, pp. 105-118, 2022, doi: 10.1007/s10666-021-09768-4.

[11] Y. Zheng, H. Zhao, and C. He, "Robust control design with optimization for uncertain mechanical systems: Fuzzy set theory and cooperative game theory," International Journal of Control, Automation and Systems, vol. 20, no. 4, pp. 1377-1392, 2022, doi: 10.1007/s12555-020-0874-y.

[12] Y. Zhang, Y. Han, X. Ji, et al., "Continuous air purification by aqueous interface filtration and absorption," Nature, vol. 610, no. 7930, pp. 74-80, 2022, doi: 10.1038/s41586-022-05124-y.

[13] A. Razek M E, M. Nasr G E, A. Baiomy M, et al., "Exhaust emissions gases effects on environmental pollution and processing technologies," Euro-Mediterranean Journal for Environmental Integration, vol. 10, no. 1, pp. 361-376, 2025, doi: 10.1007/s41207-024-00577-1.

[14] K. Patorisantis G, "Optimal control and differential game approaches in environmental and resource economics," Journal of Sustainable Energy and Environmental Development, vol. 1, no. 1, pp. 1-24, 2025.

[15] R. Cerqueti, L. Correani, and F. Di Dio, "Environmental policies in a Stackelberg differential game," Naval Research Logistics (NRL), vol. 70, no. 4, pp. 358-375, 2023, doi: 10.1002/nav.22099.

[16] K. Yuan and X. Wang, "Promoting cleaner production in ports: analysis of joint governance by central and local governments," Transportation Research Record, vol. 2678, no. 2, pp. 693-722, 2024, doi: 10.1177/03611981231175909.

[17] G. Li, S. Wu, H. You, et al., "Governments behavioral strategies in cross-regional reduction of inefficient industrial land: Learned from a tripartite evolutionary game model," Humanities and Social Sciences Communications, vol. 12, no. 1, pp. 1-17, 2025, doi: 10.1057/s41599-025-04822-y.

[18] Q. Huang and J. Shi, "Stackelberg stochastic differential games in feedback information pattern with applications," Dynamic games and applications, vol. 14, no. 5, pp. 1191-1224, 2024, doi: 10.1007/s13235-023-00549-0.

[19] U. Sadana, V. Reddy P, and G. Zaccour, "Feedback Nash equilibria in differential games with impulse control," IEEE Transactions on Automatic Control, vol. 68, no. 8, pp. 4523-4538, 2022, doi: 10.1109/TAC.2022.3206253.

[20] F. Tan and K. Qi, "Distributed non-cooperative games and distributed learning in linear and nonlinear systems: An overview," IEEE Transactions on Circuits and Systems I: Regular Papers, vol. 71, no. 8, pp. 3843-3856, 2024, doi: 10.1109/TCSI.2024.3377641.

[21] K. Ildus, P. Ovanes, and L. Yin, "Non-autonomous Linear Quadratic Non-cooperative Differential Games with Continuous Updating," Contributions to Game Theory and Management, vol. 15, no. 1, pp. 132-154, 2022, doi: 10.21638/11701/spbu31.2022.11.

[22] M. Zandebasiri, H. Jahanbazi Goujani, Y. Iranmanesh, et al., "Ecosystem services valuation: A review of concepts, systems, new issues, and considerations about pollution in ecosystem services," Environmental Science and Pollution Research, vol. 30, no. 35, pp. 83051-83070, 2023, doi: 10.1007/s11356-023-28143-2.

[23] G. Dolphin, M. Pahle, D. Burtraw, et al., "A net-zero target compels a backward induction approach to climate policy," Nature climate change, vol. 13, no. 10, pp. 1033-1041, 2023, doi: 10.1038/s41558-023-01798-y.

[24] R. Balseiro S, O. Besbes, and F. Castro, "Mechanism design under approximate incentive compatibility," Operations Research, vol. 72, no. 1, pp. 355-372, 2024, doi: 10.1287/opre.2022.2359.

[25] N. Piluso, "Is corporate social responsibility effective in improving environmental quality? Literature review," Environmental Economics, vol. 15, no. 2, pp. 1-11, 2024, doi: 10.21511/ee.15(2).2024.01.

[26] M. Fleurbaey and G. Ponthière, "The stakeholder corporation and social welfare," Journal of Political Economy, vol. 131, no. 9, pp. 2556-2594, 2023, doi: 10.1086/724318.

[27] Q. Sun, X. Wang, G. Yang, et al., "Optimal constraint following for fuzzy mechanical systems based on a time-varying β-measure and cooperative game theory," IEEE Transactions on Systems, Man, and Cybernetics: Systems, vol. 52, no. 12, pp. 7574-7587, 2022, doi: 10.1109/TSMC.2022.3160158.

[28] L. Han, J. Zhang, and H. Wang, "Shared control in pHRI: Integrating local trajectory replanning and cooperative game theory," IEEE Transactions on Robotics, vol. 41, no. 1, pp. 1263-1277, 2025, doi: 10.1109/TRO.2025.3532510.

[29] T. Bayen, A. Bouali, and L. Bourdin, "The hybrid maximum principle for optimal control problems with spatially heterogeneous dynamics is a consequence of a pontryagin maximum principle for-local solutions," SIAM Journal on Control and Optimization, vol. 62, no. 4, pp. 2412-2432, 2024, doi: 10.1137/23M155311X.

[30] R. Bonalli and B. Bonnet, "First-order Pontryagin maximum principle for risk-averse stochastic optimal control problems," SIAM Journal on Control and Optimization, vol. 61, no. 3, pp. 1881-1909, 2023, doi: 10.1137/22M1489137.

[31] B. Groom and F. Venmans, "The social value of offsets," Nature, vol. 619, no. 7971, pp. 768-773, 2023, doi: 10.1038/s41586-023-06153-x.

[32] A. Balmford, S. Keshav, F. Venmans, et al., "Realizing the social value of impermanent carbon credits," Nature Climate Change, vol. 13, no. 11, pp. 1172-1178, 2023, doi: 10.1038/s41558-023-01815-0.

[33] S. Liu and R. Yang, "Adaptive predefined-time robust control for nonlinear time-delay systems with different power Hamiltonian functions," AIMS Mathematics, vol. 8, no. 12, pp. 28153-28175, 2023, doi: 10.3934/math.20231441.

[34] L. Galimberti C, L. Furieri, L. Xu, et al., "Hamiltonian deep neural networks guaranteeing nonvanishing gradients by design," IEEE Transactions on Automatic Control, vol. 68, no. 5, pp. 3155-3162, 2023, doi: 10.1109/TAC.2023.3239430.

[35] M. Cave, "Incentive regulation: Expectations, surprises, and the road forward," Review of Industrial Organization, vol. 65, no. 2, pp. 431-453, 2024, doi: 10.1007/s11151-024-09976-8.

[36] R. Ullah, H. Ahmad, U. Rehman F, et al., "Green innovation and Sustainable Development Goals in SMEs: The moderating role of government

incentives," Journal of Economic and Administrative Sciences, vol. 39, no. 4, pp. 830-846, 2023, doi: 10.1108/JEAS-07-2021-0122.

[37] L. Rong and M. Xu, "Impact of altruistic preference and government subsidy on the multinational green supply chain under dynamic tariff," Environment, Development and Sustainability, vol. 24, no. 2, pp. 1928-1958, 2022, doi: 10.1007/s10668-021-01514-w.