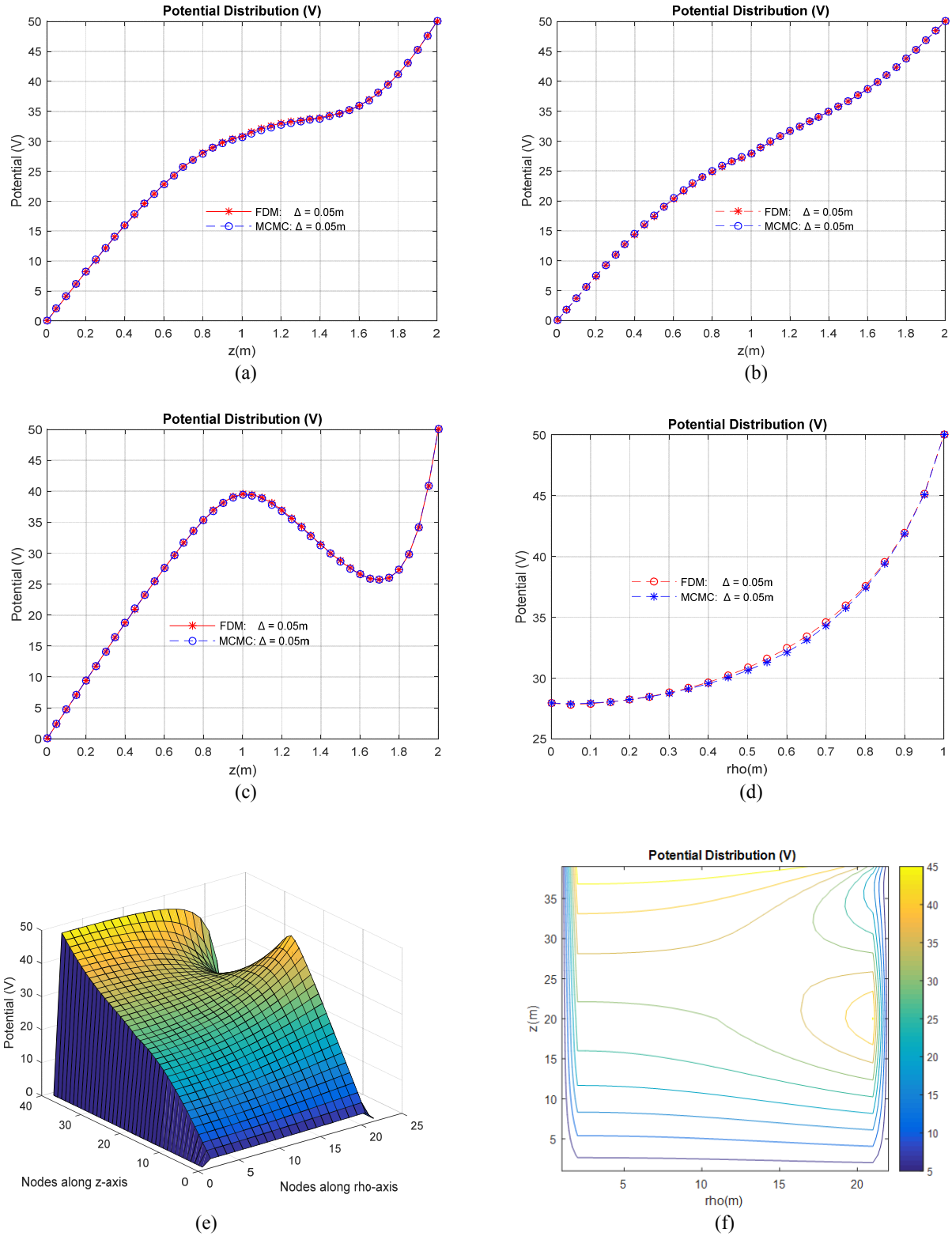

Figure 6. Potential Distribution along (a) $\rho = 0.5\text{m}, 0 \le z \le 2\text{m}$; (b) $\rho = 0, 0 \le z \le 2\text{m}$ (line of symmetry) (c) $\rho = 0.9\text{m}, 0 \le z \le 2\text{m}$ (d) $z = 1, 0 \le \rho \le 1\text{m}$ (media interface), (e) Surface Plot (f) Contour Plot for Axisymmetric Inhomogeneous Problem with mixed boundary potentials.

7. Conclusion

With growing cost of data storage and increasing complexity of electromagnetic problems facing device designers today, opportunity frequently exists to approximate problem domains to reduce problem sizes and resultant data storage requirements during implementation. Most numerical methods available nowadays involve huge iterative procedures often with large data storage requirements. The Markov Chain Monte Carlo (MCMC) solutions presented in this paper is fast and efficient in implementation once matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$ are accurately calculated. In the paper, inhomogeneous axisymmetric problems are considered. Specifically, Laplace's equations in inhomogeneous cylindrical system are solved using axisymmetric approximations. Two cases of the problem based on constant and mixed boundary potentials are considered. The MCMC solutions presented in this paper agree well with the finite difference method while using less computational resources as the method requires no iterations.

The MCMC method presented in this paper can be extended to homogeneous and inhomogeneous axisymmetric Poisson's equations as well as axisymmetric problems in spherical coordinates.

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