A New Approach to Estimate the Complex Relative Permittivity of Materials Based on Free-Space Reflection Method

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Abstract

A microwave free-space reflection method for determining the complex permittivity of planar materials is demonstrated. The improved the MUltiple SIgnal Classification (MUSIC) algorithm is used to estimate the complex relative permittivity of material based on the reflection coefficients measurement of a metal-backed material sample placed in a free-space with the thickness of the samples is the much greater than the wavelength. The measurement system consists of a pyramidal horn antenna operating at X-band and a metal-backed sample placed in a free-space. From the measured values of the reflection coefficients through a known thickness of a planar slab of the material sample, the complex relative permittivity of the material samples is estimated by the proposed algorithm without determining an integer time the wavelength in the sample under test. The proposed algorithm is verified with different thickness material samples having different thicknesses at X-band. The estimation results show that the complex relative permittivity of material samples is accurate when thickness changes.

1. Introduction

Permittivity is a physical quantity that describes how a dielectric material responds to an electric field. The knowledge of permittivity is of great importance in various areas of science and engineering. Several methods to measure the complex permittivity of dielectric materials have been developed. Among these methods, the methods for determining the parameters of material using microwave propagation in free-space are nondestructive, contactless, and sample preparation requirements are minimal. Therefore, they are especially suitable for measurement of the complex permittivity and complex permeability of materials under high-temperature conditions [1], [2]. For these reasons, they can be implemented in industrial processes where particular parameters such as density and moisture content need to be monitored in real time [3]-[5]. The free-space methods are based on the measurements of the phase of the reflection (S_{11}) and transmission (S_{21}) coefficient through a known thickness of the material samples. The reflection and transmission coefficients are determined by measuring the attenuation and phase shift introduced by a sample placed between two antennas. However, when the wavelength in the material sample is smaller than the thickness of the material under test, a phase problem is encountered [6]-[13]. Because phase-angle measurements are only possible between -1800 and +1800. The total phase shift is measured using the vector network analyzer, shifted by n times 3600, where n is an integer to be determined. This problem was solved by making measurements on samples of different thickness [14] or by the delay time if the wave is non-dispersive in the observed frequency range [15] or by the measurement technique, based on reflection, for thickness and permittivity determination [16] or by the based on selecting a sample thickness that allows the phase to remain within the measurement limits of the instrumentation at a given frequency or two frequencies [17]. On the other hands, in our previous work [18], a proposed method to estimate the complex permittivity of the material samples placed in free space is carried out by a measurement system that consists of a transmitting and a receiving horn antennas to calculate transmission coefficient (S_{12} or S_{21}) electromagnetic wave propagation in free space.

In this paper, we propose a novel approach using a high-resolution algorithm based on the improved MUSIC algorithm [19] and exploiting the relationship of the permittivity and the refractive index of the material to solve the phase ambiguity problem. Different from [18], the proposed measurement system only consists of a pyramidal horn antenna operating at 8 GHz to 12 GHz frequency range and a metal-backed sample placed in a free-space. Therefore, the values of the reflection coefficients are measured through a known thickness of a planar slab material sample. Moreover, the proposed algorithm can be to estimate the complex relative permittivity of samples when the thickness is the much greater than the wavelength in the material sample without determining an integer time the wavelength in the sample under test.

The rest of this paper is organized as follows. Section 2 describes the reflection method while estimation procedure is presented in Section 3. Section 4 is the implementation modeling by Computer Simulation Tool (CST) and the simulated estimation results. Finally, a brief conclusion is given in Section 5.

2. Free-Space reflection method

Fig.1 shows a typical setup for a free-space reflection measurement.
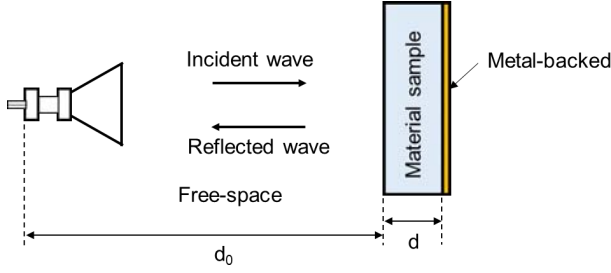

Figure 1: Free-space reflection method.

Reflection measurement of the system is only considered on D discrete reflection points which are the frequency points from f_1 to f_M within the sampling frequency range (bandwidth equivalent to $f_M - f_1$). If the number of multiple reflections is limited (and can be considered as negligible, then the number of signals is equal to D. The following formulation must also be applied to this measurement. It is assumed that the antenna has D signal sources changing from f_1 to f_M and radiating through the free-space and material sample to the metal-back then reflected back. The data obtained by the system above are the reflection coefficients at M uniformly sampled frequency points f_i ($i = 1,2,...,M : M > D$) with the same frequency Δf between adjacent points. Delay time is represented corresponding to the kth reflection point by t_k ($k = 1,2,...,D$). Then, the measurement value of reflected signals at the frequency f_i is given by

$$x_i(t) = \sum_{k=1}^{D} s_k(t)e^{-j2\pi f_i t_k} + w_i(t) \qquad (1)$$

where $s_k(t)$ is the reflection coefficient of the kth reflection point at the frequency f_i, $w_i(t)$ is the additive noise with zero mean and variance σ^2.

Fig. 1 shows a planar sample of thickness d placed in free space. The complex permittivity, relative to free space, is defined as:

$$\varepsilon^* = \varepsilon' - j\varepsilon'' \qquad (2)$$

where ε' and ε'' are the real and imaginary parts of the complex relative permittivity.

When the signal passes through the material sample, some part of it is always attenuated. To take this into account, an imaginary part is added to the refractive index. The complex refractive index (η^*) is defined as:

$$\eta^* = \eta' - j\eta'' \qquad (3)$$

where η' and η'' are the real and imaginary parts of the refractive index.

The complex relative permeability of non-magnetic materials is equal 1.0. The complex refractive index is related to the complex permittivity by the following equation:

$$\eta^* = \sqrt{\varepsilon' - j\varepsilon''} \qquad (4)$$

The time of arrival is also equal to delay time ($\tau = t_k$) and it is determined by

$$\tau = 2\left(d\frac{\sqrt{\varepsilon' - j\varepsilon''}}{c} + d_0\frac{1}{c} \right) \qquad (5)$$

where d_0 is the distance between the port of antenna and material sample, c is the light velocity in free-space.

The difference in phase ($\Delta\varphi_i$) of the signal when it goes through the medium (free space and material sample) at the first frequency and the ith frequency is:

$$\Delta\varphi_i = \frac{4\pi(i-1)\Delta f}{c}\left(d\sqrt{\varepsilon' - j\varepsilon''} + d_0 \right) \qquad (6)$$

Combine this with the phase difference, the relation (1) can be written using the vector notation as follows:

$$X = AS + W \qquad (7)$$

where:

$X = [x_1(t), x_2(t), ... x_M(t)]^T$, X is the Mx1 output vector measured at receiver, T denotes transpose.

$S = [s_1(t), s_2(t), ..., s_D(t)]^T$, S is the vector of the k arriving signals.

$W = [w_1(t), w_2(t), ..., w_M(t)]^T$, W is the noise vector:

$A = [a(\varepsilon_1', \varepsilon_1''), a(\varepsilon_2', \varepsilon_2''), ..., a((\varepsilon_D', \varepsilon_D''))]$, A is an MxD "parameter" matrix with $a(\varepsilon_k', \varepsilon_k'') = [e^{-j\Delta\varphi_1}, e^{-j\Delta\varphi_2}, ..., e^{-j\Delta\varphi_i}]^T$ is a "parameter" vector of each signal.

3. Complex relative permittivity estimation procedure

The multiple signal classification algorithm was proposed by R. Schmidt [19]. The basic approach of this algorithm is that from the received signal, the covariance matrix is calculated and then eigenvectors decomposition is carried out. The signal subspace and noise subspace are determined based on eigenvectors and eigenvalues. The results showed that the $M - D$ dimensional subspace spanned by the $M - D$ noise eigenvectors as the noise subspace and the D dimensional subspace spanned by the incident signal parameter vectors as the signal subspace; they are disjoint. The signal and the

noise subspaces are calculated by the matrix algebra and they are found to be orthogonal to each other. Therefore, the signal and noise subspaces are isolated by the orthogonal property of this algorithm. Thus, the complex relative permittivity of the material sample is estimated by combining the autocorrelation and MUSIC function of the received signal.

The corresponding data covariance matrix in (7) is given by

$$R_{XX} = E\{XX^H\} = AS_{XX}A^H + \sigma^2 I \quad (8)$$

where $S_{XX} = E\{SS^H\}$ denotes the signal covariance matrix, I is the identity matrix, H denotes complex conjugate transpose.

The eigenvalues of R_{XX} are $\lambda_1, \lambda_2, \ldots, \lambda_D$ such that:

$$det(R_{XX} - \lambda_i I) = 0 \quad (9)$$

Substituting (8) to (9):

$$det(AS_{XX}A^H - (\lambda_i - \sigma^2)I) = 0 \quad (10)$$

The eigenvalues ξ_i of $AS_{XX}A^H$ are:

$$\xi_i = \lambda_i - \sigma^2 \quad (11)$$

If the eigenvalues ξ_i of $AS_{XX}A^H$ are zero, $AS_{XX}A^H$ is singular. This means that the number of incident wave fronts D is less than the number of frequency elements M. Thus, The minimum eigenvalue of R_{XX} is equivalent to σ^2 with multiplicity $M - D$. Therefore:

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_D \succ \lambda_{D+1} = \lambda_{D+2} = \cdots = \lambda_M = \sigma^2 \quad (12)$$

The eigenvector u_i associated with the eigenvalue λ_i satisfies the following equation:

$$(R_{XX} - \lambda_i I)u_i = 0 \quad (13)$$

For eigenvectors associated with the minimum eigenvalue, the (14) is suggested by substituting (8) and (12) into (13).

$$AS_{XX}A^H u_i = 0 \quad (14)$$

Since A has full rank and S_{XX} is non-singular, thus:

$$A^H u_i = 0 \quad (15)$$

This means that the eigenvectors corresponding to the minimum eigenvalue are orthogonal to the columns of the matrix A. Namely, they are orthogonal to the "parameter" vector of the signals: $\{u_{D+1}, \ldots, u_M\} \perp \{a(\varepsilon_1', \varepsilon_1''), \ldots, a(\varepsilon_D', \varepsilon_D'')\}$.

It implies that the squared norm of $A^H u_i$ is zero

$$\|A^H u_i\|^2 = a^H(\varepsilon', \varepsilon'')U_M U_M^H a(\varepsilon', \varepsilon'') = 0 \quad (16)$$

where $U_M = [u_{D+1}, u_{D+2}, \ldots, u_M]$ represents the eigenvectors associated with the noise subspace of the covariance matrix R_{XX}.

The pseudo-spectrum of the MUSIC function as (17) is given by combining the autocorrelation function of signal subspace:

$$P_{MUSIC}(\varepsilon', \varepsilon'') = \frac{a^H(\varepsilon', \varepsilon'')a(\varepsilon', \varepsilon'')}{a^H(\varepsilon', \varepsilon'')U_M U_M^H a(\varepsilon', \varepsilon'')} \quad (17)$$

The values of ε' and ε'' that make P_{MUSIC} reach a peak that are chosen from the result of the estimation.

4. Implementation modeling and simulation results

4.1. System modeling

In order to estimate the reflection coefficients (S_{11}) for the free-space reflection method presented in section II, in this part, we have implemented the model using CST software, the flowchart and numerical setup performs modeling to determine parameter S_{11} as shown in Figure 2.
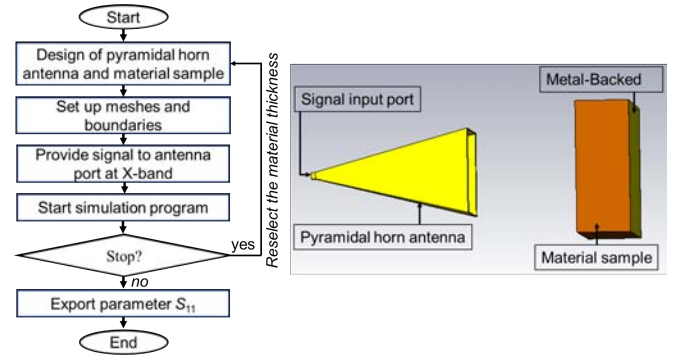

Figure 2: Modeling to determine parameter S_{11}: Left (Flowchart) and Right (Numerical setup).

In Fig. 2, one pyramidal horn antenna is designed to operate well in the frequency range of 8.0 – 12.0 GHz [20]. The gain and voltage standing wave ratio of the pyramidal horn antenna are 20 dBi and 1.15 at center frequency. In this model, the distance between the port of antenna and the material sample is 1082.5mm. The calibration procedure requires to withdraw the losses due to free-space, this needing to proceed to the same measurements with air material. Table 1 shows the complex relative permittivity values of material samples at 10 GHz.

Table 1: The parameters of material samples.

Material	ε'	ε''	Thickness d (mm)
FR-4 (Lossy)	4.3	0.025	10 ÷ 90
RO4350B (Lossy)	3.66	0.0037	10 ÷ 90
TEFLON-PTFE (Lossy)	2.1	0.0002	10 ÷ 90

4.2. Simulation results

The material samples are measured at 801 different frequencies from 8.0 to 12.0 GHz with the scale of 5 MHz. The simulation was carried out on PCs with Intel Core i3 of memory chips, at 2.2 GHz speed and 8G RAM. The total simulation and processing time for one example is about 2 hours' real time. From Fig. 3 to Fig. 7 show the pseudo-spectrum for the permittivity of FR-4 sample with thickness of 10 mm, 30mm, 50 mm, 70 mm and 90 mm, respectively.
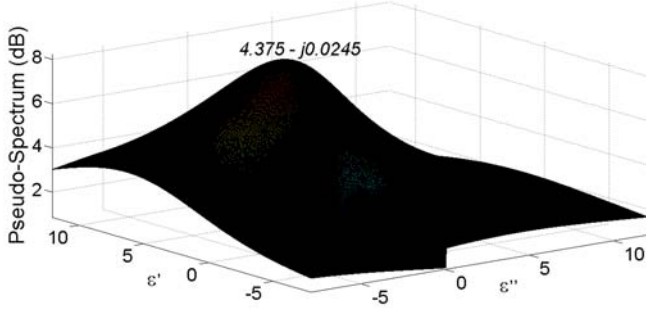

Figure 3: Pseudo-spectrum of FR-4 sample at 801 frequencies, frequency range 4.0 GHz and thickness 10 mm.
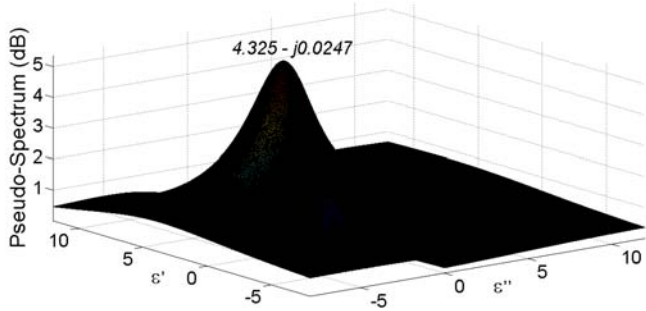
Figure 4: Pseudo-spectrum of FR-4 sample at 801 frequencies, frequency range 4.0 GHz and thickness 30 mm.
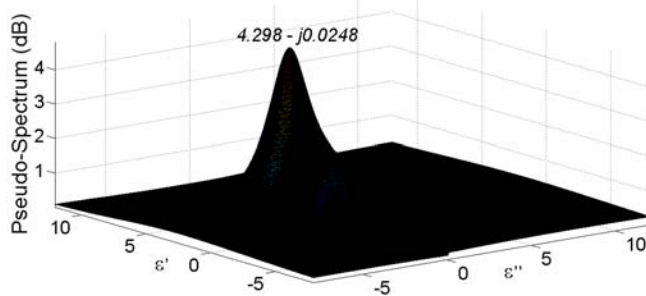

Figure 5: Pseudo-spectrum of FR-4 sample at 801 frequencies, frequency range 4.0 GHz and thickness 50 mm.
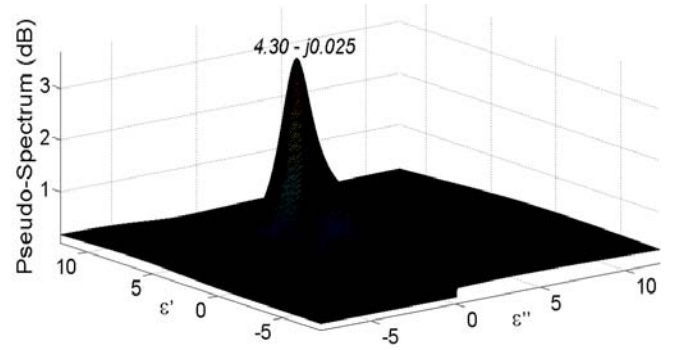

Figure 6: Pseudo-spectrum of FR-4 sample at 801 frequencies, frequency range 4.0 GHz and thickness 70 mm.
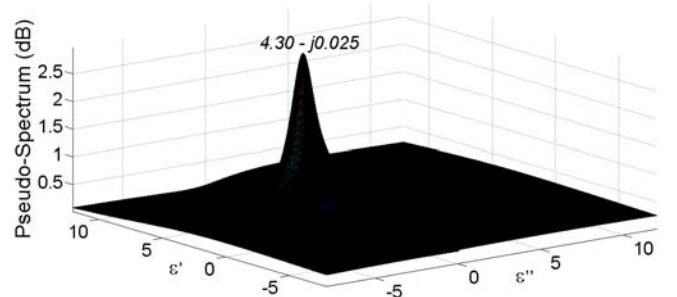

Figure 7: Pseudo-spectrum of FR-4 sample at 801 frequencies, frequency range 4.0 GHz and thickness 90 mm.

It can be seen that from Fig. 3, 4, 5, 6 and 7, respectively, the change in the thickness of the sample affects both the sharpness and the position of the peak of pseudo-spectrum. The change in the position of the peak from the expected value means that the estimation is not accurate for materials with small thickness. Furthermore, the drop in sharpness of the peak makes the determination of the point of maximum spectrum more difficult because the points around the peak can become equal to or greater than the supposed peak. These changes combined create a sharp decrease of the accuracy of the results as the thickness of the samples decreases.
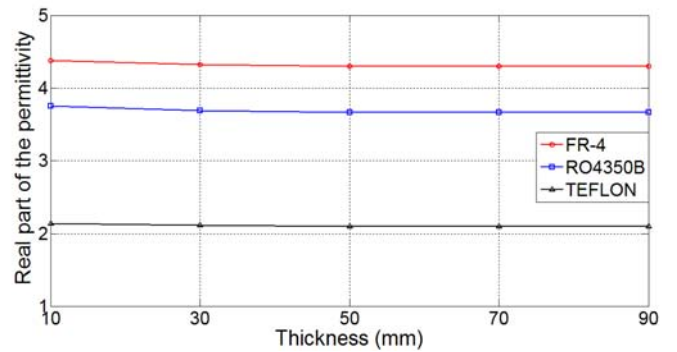

Figure 8: The real part of the complex relative permittivity of material samples is estimated by the MUSIC algorithm.
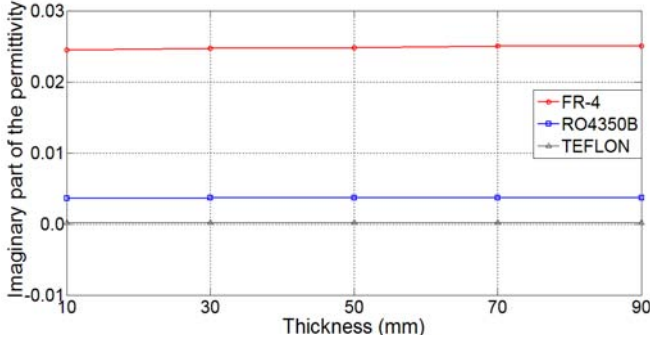
Figure 9: The imaginary part of the complex relative permittivity of material samples is estimated by the MUSIC algorithm.
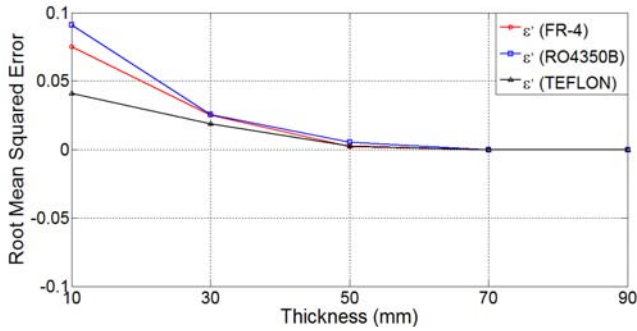

Figure 10: Root mean squared error versus thickness graph for ε'.
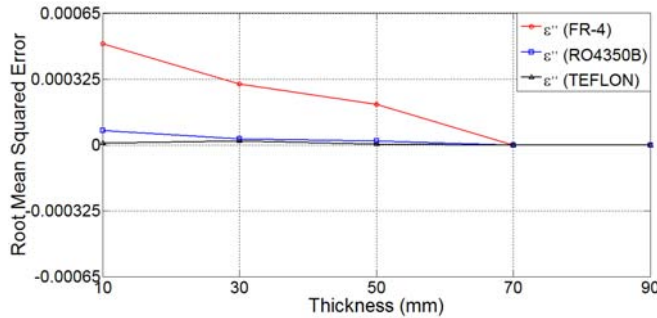

Figure 11: Root mean squared error versus thickness graph for ε''.

The estimation results show that the complex relative permittivity of material samples is accurately when thickness changes as Fig. 8 and Fig. 9, Fig. 10 and Fig. 11, respectively, show the root mean squared error (RMSE) versus the thickness graph calculated from the simulation results for the three samples at different thicknesses. From the results, we can see that the algorithm can solve for the unique value of the permittivity regardless of the thickness d, but is more accurate with samples of larger thickness. The thickness of the sample affects the accuracy of the measurement of both ε' and ε''. To get the accurate value of ε', the thickness d needs to be approximately 50 mm or higher, whereas it needs to be around 70 mm to get accurate results of both ε' and ε''.

5. Conclusions

A super high-resolution algorithm to estimate the complex relative permittivity of the planar material samples using free-space reflection method is proposed. The system model consists of a pyramidal horn antenna and a metal-backed material sample placed in a free-space. The parameter vectors of the MUSIC algorithm describe the difference in phase, which indicates the difference in frequencies and arrival time of the simulated signals. These parameter vectors are calculated by using the relation between the permittivity and the refractive index. The performance of the proposed algorithm is verified for all scenarios in the simulation. Through the results, the ability of the proposed algorithm to solve the problem of ambiguity of the conventional method is also validated. The measurement of the real part of the complex relative permittivity using this algorithm is accurately when the thickness of the material is at least 50 mm. Whereas, the measurement of the imaginary part of the complex relative permittivity using this algorithm is accurate when the thickness of the material is at least 70mm. The proposed algorithm has great benefits for determining the characteristic parameters of new materials.

Acknowledgements

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